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CHROMATIC AND SPHERICAL ABERRATION
IN ELECTROSTATIC LENSES

ORLAN R. CARDEN, JR.
and
GORDON W. TINKER

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IN ELECTROSTATIC LENSES

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by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
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ABSTRACT

High resolution cathode-ray display devices demand focusing systems that reduce total aberration to the lowest possible value. Magnetic focusing systems have proven superior to electrostatic systems with respect to total aberration, but their size, weight, and power requirements restrict their use. Hence considerable effort is being put into high resolution electrostatic focusing systems. The two limiting aberrations, spherical and chromatic, are analyzed in this thesis for four types of electrostatic lenses and the results compared.

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CHAPTER I

INTRODUCTION

There are three general categories of image defects which occur in electron lenses of cathode ray devices. These defects are: space charge defects, geometric aberration, and chromatic aberration.

Space charge defects are a result of a spreading of the electron beam due to mutual repulsion of electrons within the beam. This effect can be made negligible in focusing systems by maintaining the current in the beam at low levels and by placing the focusing system at a position of low energy level in the beam.¹

Geometric aberration is composed of five components: spherical aberration, coma, astigmatism, curvature of field, and distortion. In magnetic lenses, additionally, the anisotropic aberrations are present, which are rotational modifications of the above. The details of each of these components can be found in the literature.² If the electron source of the beam is lined up properly on the beam axis, we may consider the small object, which is

¹V. K. Zworykin, G. A. Morton, E. G. Ramberg, J. Hillier, and A. W. Vance, Electron Optics and the Electron Microscope (John Wiley & Sons, 1945), Section 1.6, p. 27.

²V. K. Zworykin and G. A. Morton, Television. 2nd ed. (John Wiley & Sons, 1954), Section 4.14, pp. 161-162.

the electron trajectory crossover prior to the lens, to be wholly on the axis. Under these paraxial conditions, all of the geometric aberrations except for spherical aberration, which exists even on the axis can be eliminated.³

The last of these major categories, chromatic aberration, results essentially from the presence of electrons possessing different initial velocities when emitted from a thermionic cathode. Chromatic aberration cannot be eliminated in systems employing only axially symmetric lenses operating at moderate potentials.⁴ It is noteworthy that through the use of high voltages and resulting relativistic velocities, chromatic aberration can be cancelled.⁵ Also, by superposition of electric and magnetic fields, chromatic and spherical aberration can be reduced to negligible levels. [7]

Thus we see that two types of aberration are present in, and cannot be eliminated from, typical axially symmetric electron lenses such as the type customarily utilized in the majority of cathode ray display devices.

³D. W. Epstein, "Electron Optical System of Two Cylinders as Applied to Cathode Ray Tubes," Proceedings of the Institute of Radio Engineers, v. 24 (August, 1936), pp. 1131-1132.

⁴Zworykin and Morton, op. cit., Section 4.14, p. 166.

⁵Klemperer, Electron Optics (Cambridge University Press, 1953), Section 7.3, p. 191.

These two aberrations, spherical and chromatic, constitute the topic of study for this thesis.

There are numerous types of electrode configurations which have been designed for use in the electrostatic electron lens. There are some configurations which theoretically reduce spherical aberration to negligible levels. One of these lenses is shown below in figure I-1.^[2] There are practical fabrication problems associated with this type of configuration similar to those encountered in lining up complex systems that do not possess axial symmetry. In addition to the five components of geometric

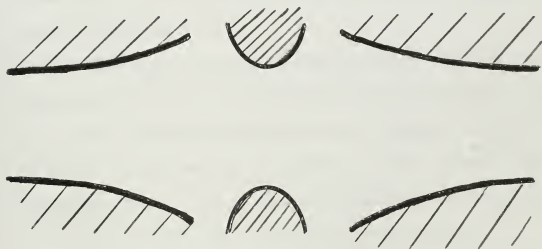


Fig. I-1 Optimum System of Minimum Spherical Aberration

aberration and the two forms of chromatic aberration, aberrations may be introduced by producing non-symmetrical fields which result from poor manufacturing techniques

and construction methods.⁶

Due to the above problems, symmetrical electrode configurations of simplicity in design and construction are utilized in industry today. Practical systems consisting of simple cylinders and planer elements are studied here for which experimental field data was obtained and aberration calculations made.

The four basic lens configurations studied are shown in figure I-2. The simplest of the electrode configurations is found in System A which is known as the Equidiameter Cylinder Immersion Lens. To provide smoother transition at points of discontinuity to prevent high voltage breakdown, the cylinder ends are sometimes flared as shown in System A-1.

A practical saving in construction cost can be obtained by increasing the diameter of the second cylinder until the coated inner wall of the cathode ray tube can be used. This results in System B, the Coaxial Cylinder Immersion Lens. Also, the cylinder ends of the first cylinder can be flared, resulting in System B-1.

By placing an aperture comprised of a conduction plane with a pin hole on the axis between the two equidiameter cylinders of System A, the result is the Aperture

⁶Epstein, op. cit., p. 1129.

Lens of System C. In this lens, the presence of the aperture results in a virtual object which reduces spot size, but also results in a reduction in beam power.⁷

By introducing a third cylinder between the two equidiameter cylinders, a lens known as the Saddle Field Lens results. A special form of this lens results when cylinders one and three are operated at the same potential. This lens is known as the Einzel Lens. When the center electrode is grounded, this lens has the advantage that its unipotential symmetric nature allows the magnification to be independent of voltage fluctuations in the power supply, and consequently the irregular changes in velocity and resulting chromatic aberration are eliminated.⁸

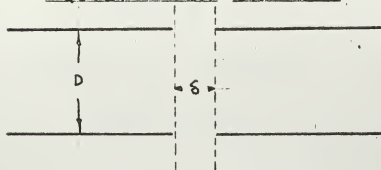
⁷V. E. Cosslett, Introduction to Electron Optics (Oxford University Press, 1953), Section 25, P. 56.

⁸Ibid., Section 26, p. 68.

SYSTEM A

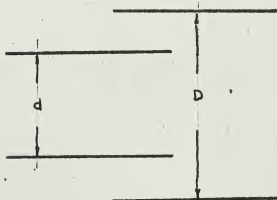
FIG. I-2

EQUIDIAMETER CYLINDER

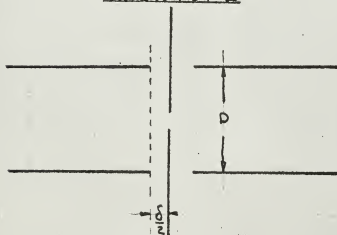


SYSTEM B

COAXIAL CYLINDER



SYSTEM C
APERTURE



SYSTEM D

EINZEL

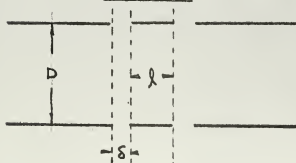
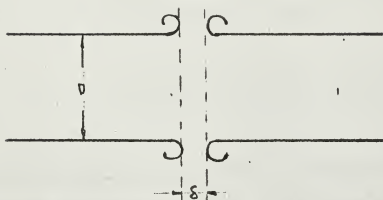


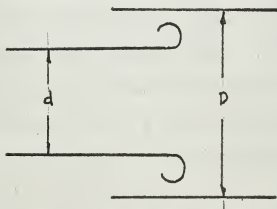
FIG. I-3

MODIFIED SYSTEMS

SYSTEM A-1



SYSTEM B-1



CHAPTER II

EXPERIMENTAL DATA

By applying Newton's Second Law of motion to the electron as a particle, the general ray equation can be derived which describes the path or trajectory of an electron through an electrostatic field.⁹

$$r'' = \frac{(1 + r'^2)}{2\phi} \left[\frac{\partial \phi}{\partial r} - r' \frac{\partial \phi}{\partial z} \right]$$

The above equation is written in cylindrical coordinates which can be readily applied to axially symmetric systems. The variables (r, z) define the point on the trajectory and the potential at the point is defined by ϕ .

If we assume that a physical aperture exists at the cathode end of the focusing system which limits the cross section of the beam to a small diameter and if only rays which made small angles with the axis are considered, then we may postulate that the electron remains close to the axis throughout its trajectory through the field.¹⁰ Under these conditions ϕ may be replaced by a power series ex-

⁹Zworykin et al., op. cit., Chapter 12, pp. 401-402.

¹⁰J. R. Pierce, Theory and Design of Electron Beams (D. Van Nostrand Co., 1954), Section 8.6, p. 135.

pansion along the axis which defines the field throughout the region through which all trajectories pass.¹¹ By additionally neglecting all terms of the expansion of higher order than the first in r and r' , the paraxial ray equation is derived.

$$r'' + r' \frac{\phi'}{2\phi} + r \frac{\phi''}{4\phi} = 0$$

It is through the use of the above equation and associated assumptions that chromatic and spherical aberration will be studied. The method used herein for the study of chromatic aberration is one of ray tracing utilizing the paraxial ray equation. The technique employed in the study of spherical aberration differs in that it requires, in addition to the paraxial solution of two primary rays, the calculation of a complicated integral. This method was derived by Zworykin.¹²

In order to accomplish the above, the potential ϕ at intervals along the axis must be obtained for the configuration to be studied. There are numerous methods which have been derived for obtaining this potential data both analytically and experimentally. Among the analytical

¹¹Zworykin et al., op. cit., Chapter 11.6, p. 375.

¹²Zworykin et al., op. cit., Section 16.3, p. 555.

methods are the "relaxation method"¹³ which is essentially a method of successive approximations and a numerical analysis approach to solve LaPlace's equation with boundary conditions.¹⁴ Strict analytical solution of the field is highly complex and little effort along this line has been made. [7]

Other techniques worthy of mention are of a so-called semi-analytical type. A method developed by Bertram [4] employs the experimental measurement of potentials at a radius from the axis and then through the use of Bessel functions the voltages along the axis are found by extrapolation. This method may prove to be superior to those considered, for it is in the area of axis measurement where some problems were encountered in the work of this thesis.

We obtained axial potential data from an analog resistor network where the boundary conditions established by the electrode configurations were set up. The method and the resistor network is described in a paper by its designer, J. R. Hechtel. [11, 12] In the same paper, the designer established the accuracy of the network which is acceptable and equivalent to other methods. Potentials

¹³Cosslett, op. cit., Chapter 2, Section 13, p. 22.

¹⁴Zworykin et al., op. cit., Section 11.6, p. 375.

applied to the resistor board were 0 volts and 10 volts such that readings taken were in normalized form to be later used with any voltage ratio.¹⁵

There are two techniques for taking the measurements of axial potential from the resistor board. The first of these methods utilized a digital voltmeter with input impedance of 15 megohms. The second method employs a bridge and galvanometer null technique. The digital voltmeter proved to be sufficiently accurate except in the fourth place. It was found that since the resistors on the axis were 200 K-ohms, the readings in the fourth place could not be trusted. It was observed that as the readings were taken at some distance off the axis, the accuracy increased since the values of the resistors are not the same throughout the network and actually range from 200K on the axis to 1K at 25 nodes off the axis.^[2] We believe that through the use of Bertram's method, some improvement in accuracy could be effected, since off axis measurements do not load down the resistor board as much as those on the axis. Since the resistor analog network utilized was in great demand for current work of the owner-manufacturer,¹⁶ the first method was employed to avoid the necessity of the

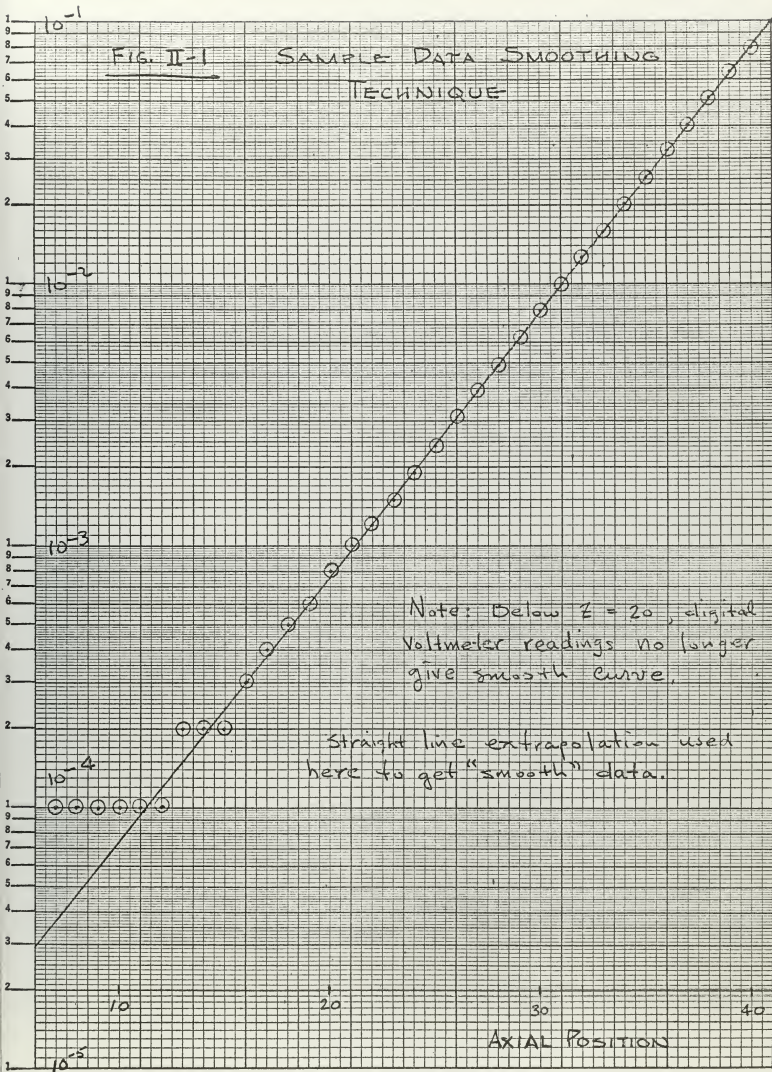
¹⁵Klemperer, op. cit., Section 3.2, p. 32.

¹⁶Litton Industries, San Carlos, California.

use of the extremely slow bridge method for low voltage readings. We found that computer analysis required not only a smooth potential function but also smooth first and second derivatives of the axial potential. We noted that approximations to Immersion Lens fields contain the TANH function. By plotting on semilog graph paper the voltage readings along the axis, we found that a straight line resulted when not near the center of the lens, which showed deviations only in the low voltage region where the digital voltmeter limitations were reached. The straight line was used to extrapolate the end portions of the curve. This technique was employed throughout the studies undertaken. Figure II-1 shows an example. All smooth data used is tabulated in Appendix I for the nineteen lenses studied.

FIG. II-1

SAMPLE DATA SMOOTHING TECHNIQUE



CHAPTER III

OPTICAL PROPERTIES OF LENSES

Comparison of lens qualities could not be done without looking at the focal properties and location of the cardinal points of each lens. [1, 2] This chapter is a series of graphs that pictorially show the relative strengths of the nineteen lenses studied. Two types of curves are plotted: 1) Normalized potential data vs. Axial distance; and 2) focal length and mid-focal length in units of diameters vs. Voltage Parameter.

Program SPHERAX was geared to provide this data along with the calculation of spherical aberration. Figure III-1 shows the definitions of parameters used. Note that there

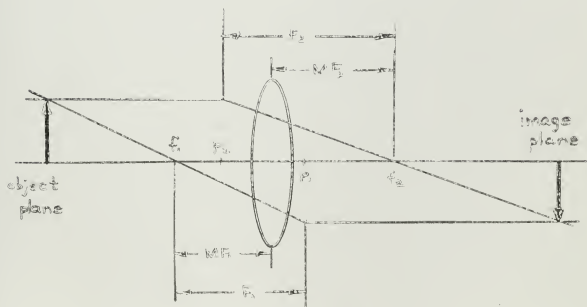


Fig. III-1 Principal Rays of Simple Lens and Location of Cardinal Points

is only one curve of potential data plotted for the two cylinder immersion lenses. The data for the six lenses studied does not vary sufficiently to be seen on this scale.

FIG. III-2

EQUIDIAMETER CYLINDER IMMERSION LENS

AXIAL VOLTAGE DATA

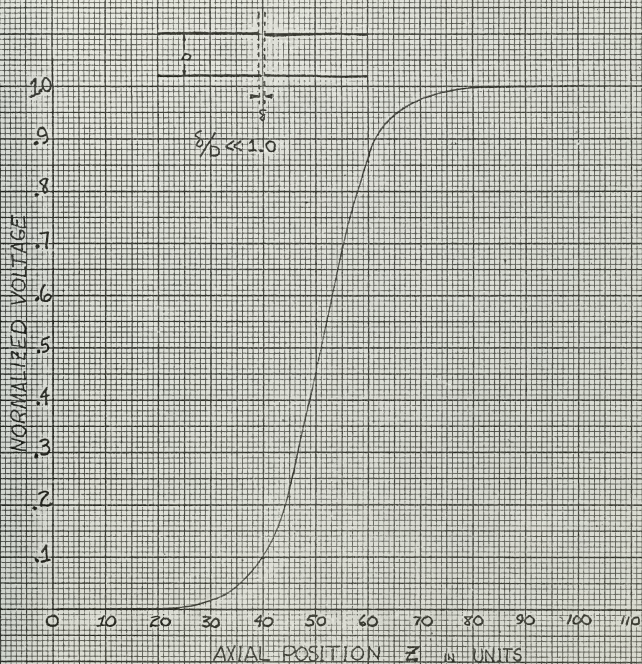


FIG III-3

COAXIAL CYLINDER
(OVERLAPPED)
IMMERSION LENS

NORMALIZED VOLTAGE

AXIAL POSITION z IN UNITS

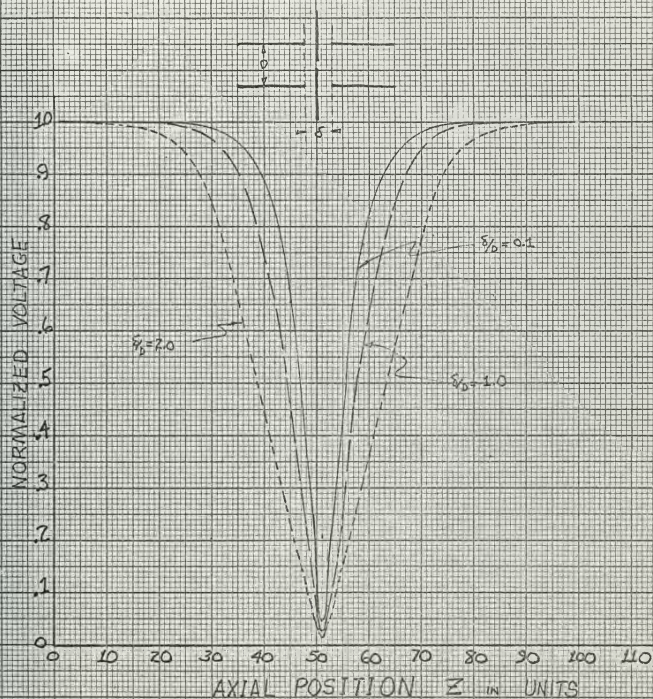
$\rho_0 = 2.0$
FLARED

$L, \rho_0 = 2.0$

$\rho_0 = 1.5$

$\rho_0 = 2.5$

FIG. III-4

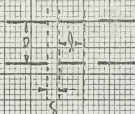
APERTURE LENS
AXIAL VOLTAGE DATA

AXIAL VOLTAGE DATA

FIG. III-5

FINZEL LENS

SPACING TO DIAMETER RATIO = 0.6



$g/D = 0.6$

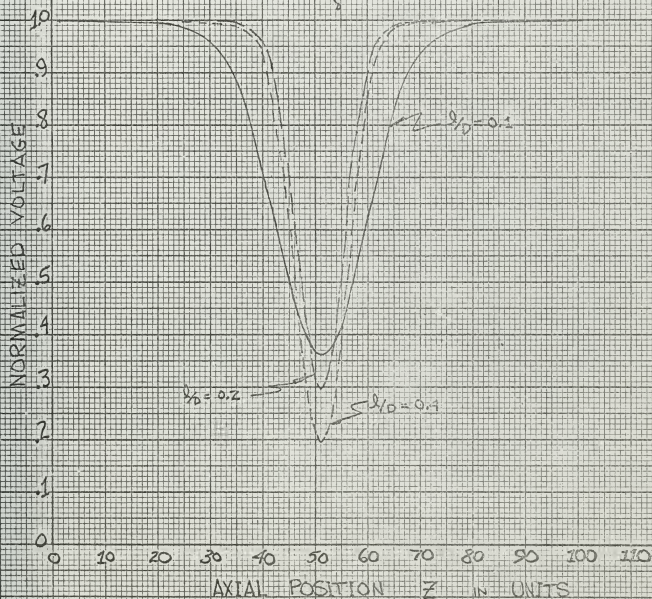


FIG. III-6

AXIAL VOLTAGE DATA

EINZEL LENS

SPACING TO DIAMETER RATIO $\ll 1.0$

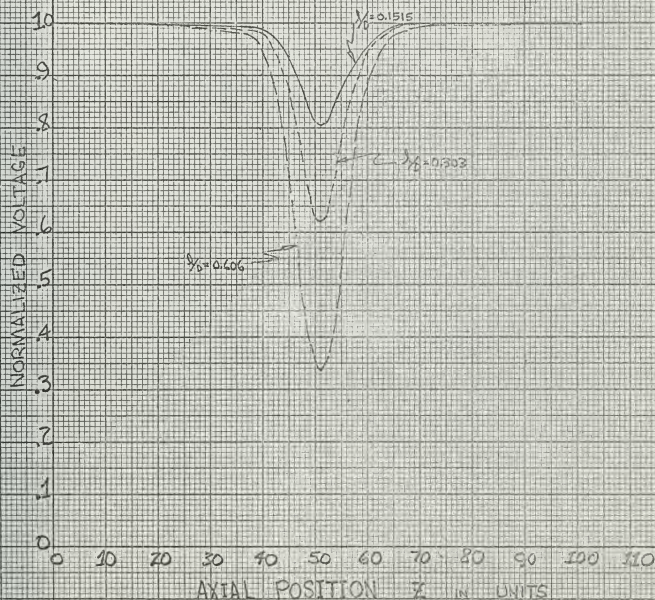


FIG. III-7

FOCAL PROPERTIES OF THE EQUIDIAMETER CYLINDER LENS (WIDELY SPACED)

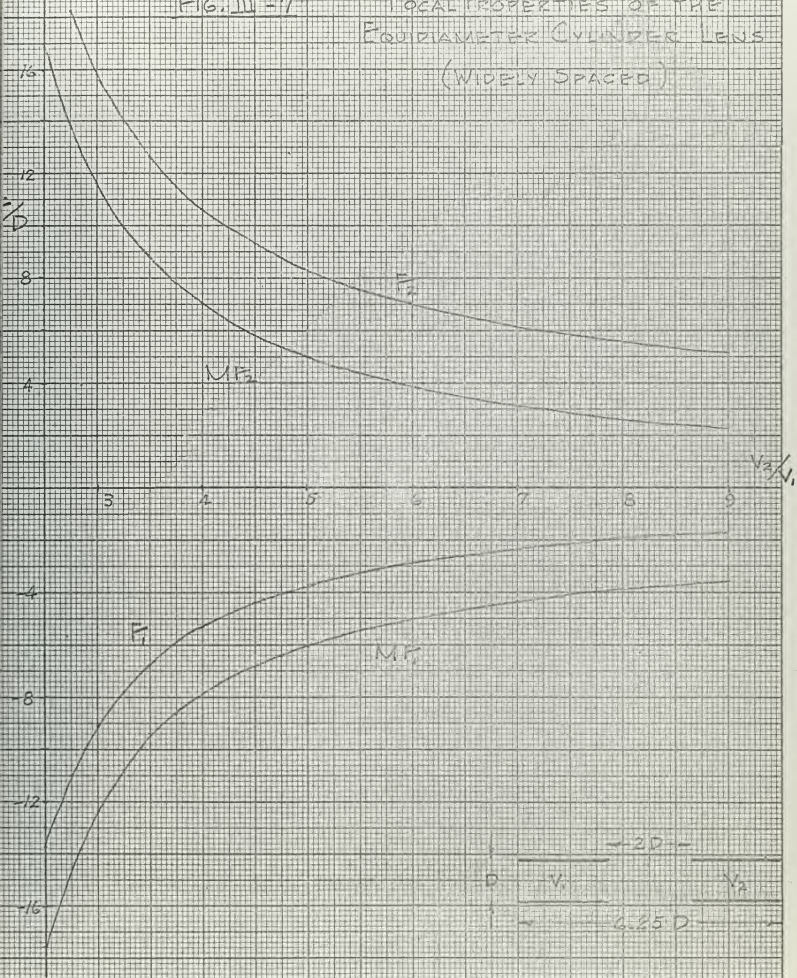


FIG. III-8 FOCAL PROPERTIES OF THE
EQUIDIAMETER CYLINDER LENS
(MEDIUM SPACING)

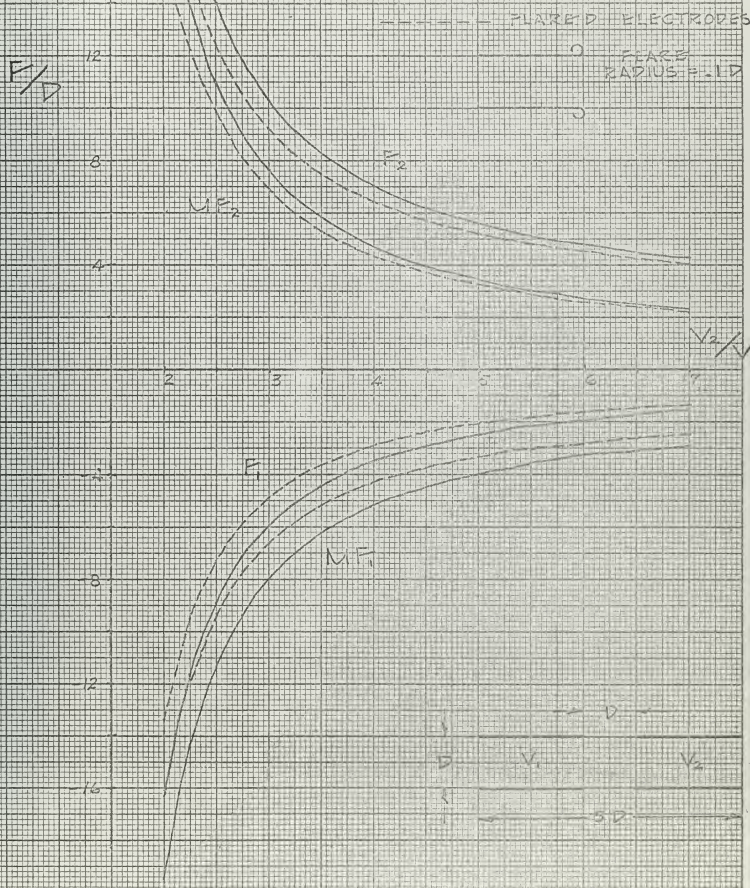


FIG. II-9

FOCAL PROPERTIES OF THE EQUIDIAMETER CYLINDER LENS (MEDIUM SPACING)

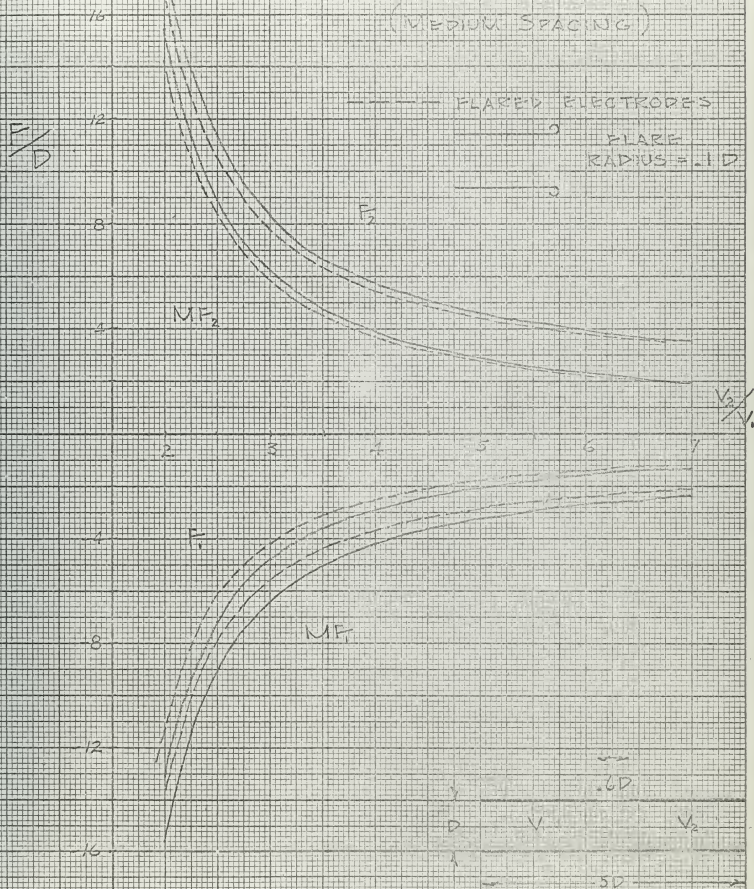


FIG. III-10

FOCAL PROPERTIES OF THE
EQUIDIAMETER CYLINDER LENS
(CLOSELY SPACED)

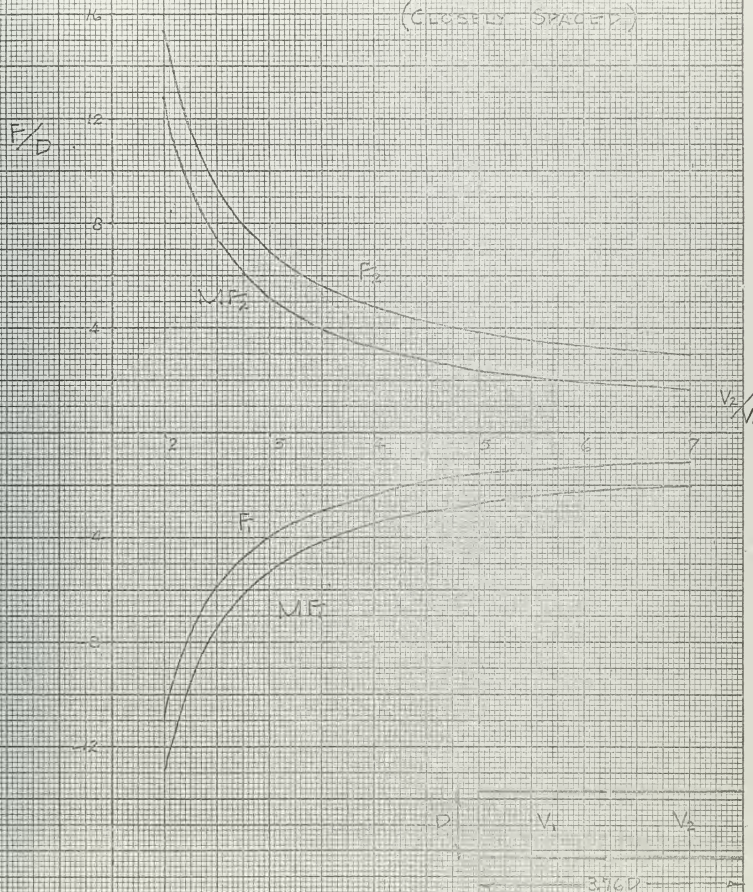


FIG. III-11

FOCAL PROPERTIES OF THE COAXIAL CYLINDER LENS

$$\frac{D_2}{D_1} = 1.5$$

$\frac{F}{D}$

16

12

8

4

MF_2

F_2

2

3

4

5

6

7

$\sqrt{2}/\sqrt{1}$

4

F_1

MF_1

8

12

16

D_1

$1.5 D_1$

$2.5 D_1$

FIG II-12 FOCAL PROPERTIES OF THE COAXIAL CYLINDER LENS

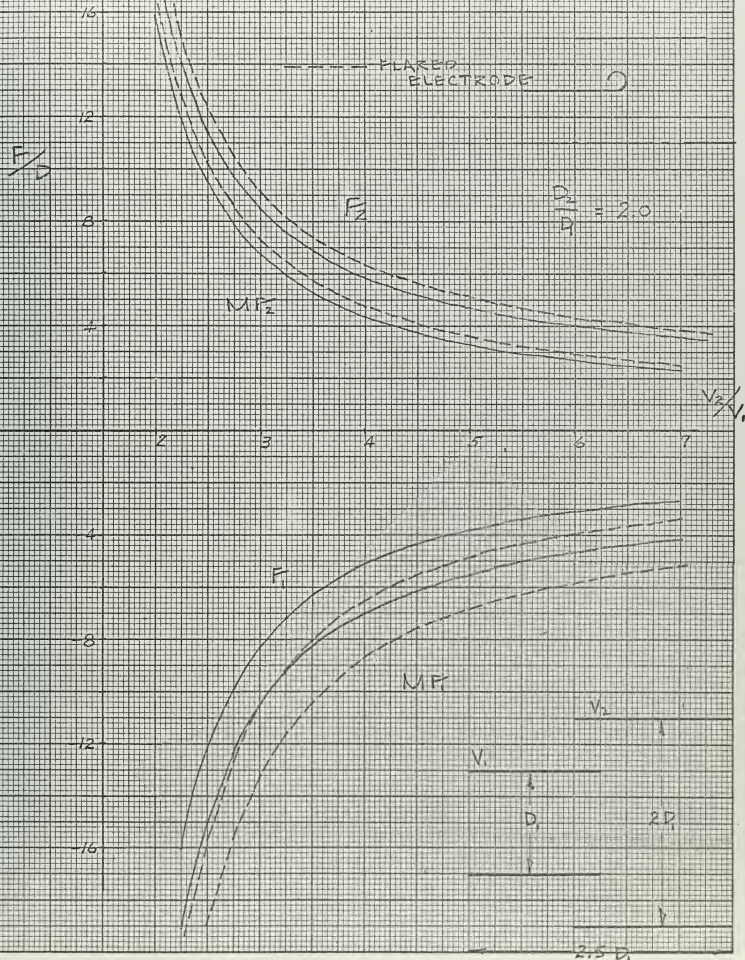


FIG. III-13

FOCAL PROPERTIES OF THE COAXIAL CYLINDER LENS

$$\frac{D_2}{D_1} = 2.5$$

F/D

16

12

8

4

F_2

MF_2

V_2/V_1

2

3

4

5

6

7

4

-8

-12

-16

F_1

MF_1



$2.5D_1$

$2.5D_1$

FIG III-14

FOCAL PROPERTIES OF APERTURE LENSES

F/D

APERTURE DIAM. = 1.0

$\delta = 2.0$

$\delta = 0$

$\delta = 1.0$

F

MP

V_2/V_1

INITIALLY ACCELERATING FIELD

V_2

V_1

V_1

D

1.0

δ

δ

F/D

$\delta = 2.0$

$\delta = 0$

$\delta = 1.0$

F

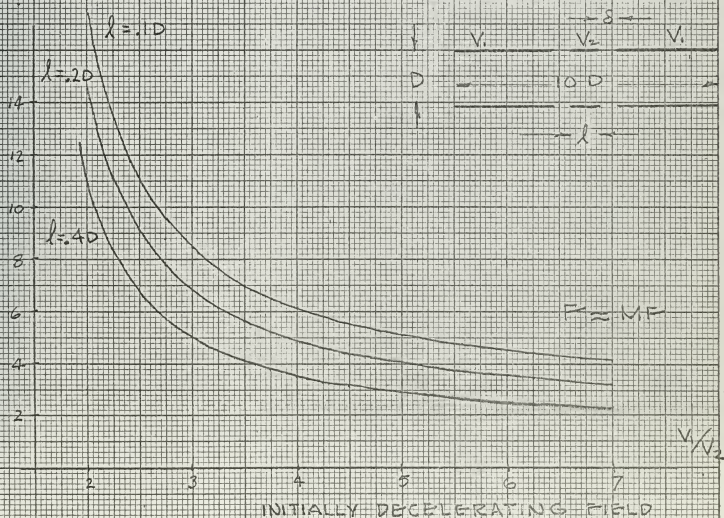
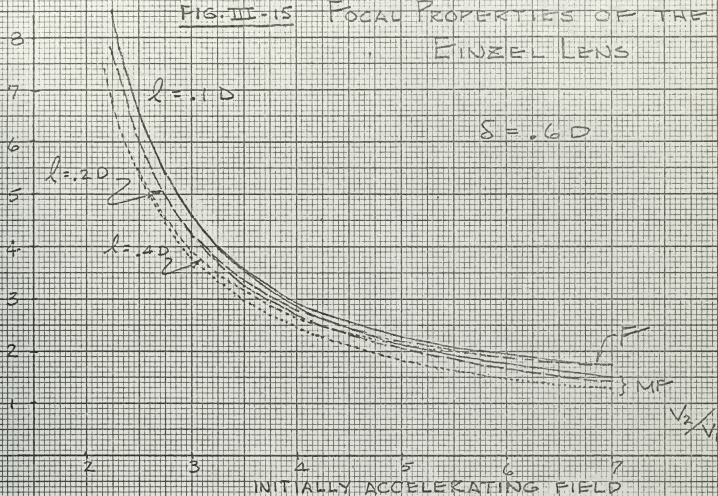
MP

V_1/V_2

INITIALLY DECELERATING FIELD

FIG. III-15

FOCAL PROPERTIES OF THE EINZEL LENS

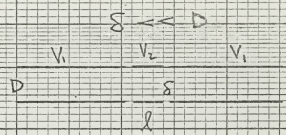


F/D

FIG. III-16

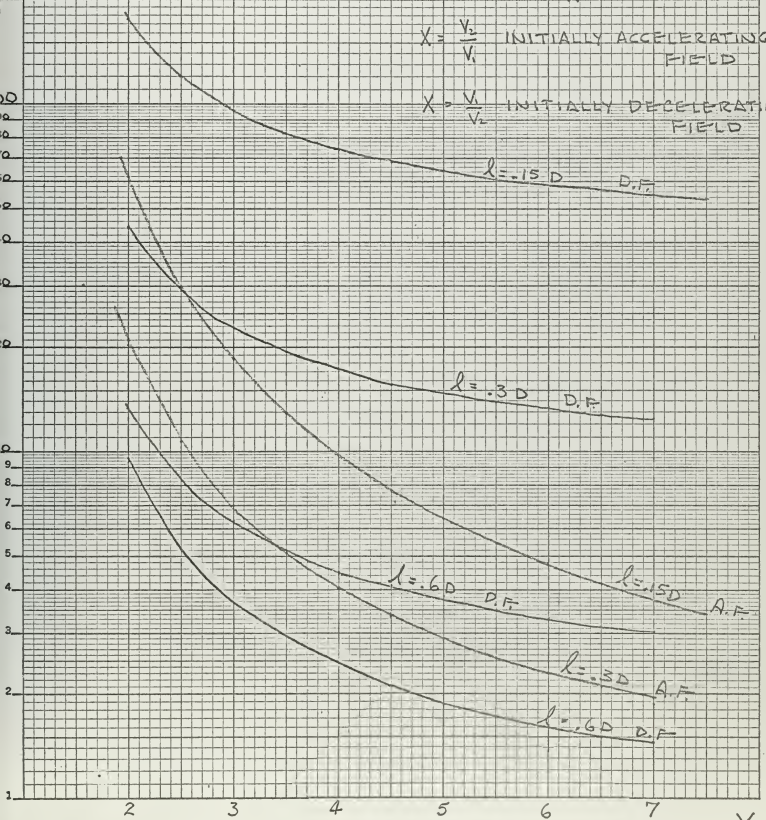
FOCAL PROPERTIES OF THE EINZEL LENS

$F \approx MF$



$X = \frac{V_2}{V_1}$ INITIALLY ACCELERATING FIELD

$X = \frac{V_1}{V_2}$ INITIALLY DECELERATING FIELD



CHAPTER IV

CHROMATIC ABERRATION

Chromatic Aberration results from inhomogeneities in the initial kinetic energy of electrons leaving a thermionic cathode. Initial velocities are distributed according to the well-known Maxwell-Boltzmann distribution:

$$n(v) dv = \left(\frac{e}{kT} \right)^2 \exp \left(- \frac{eV}{kT} \right) dv$$

where: k = Boltzmann's constant = 1.38×10^{-16} erg/degree
 T = Absolute temperature of cathode
 eV = initial kinetic energy in electron volts

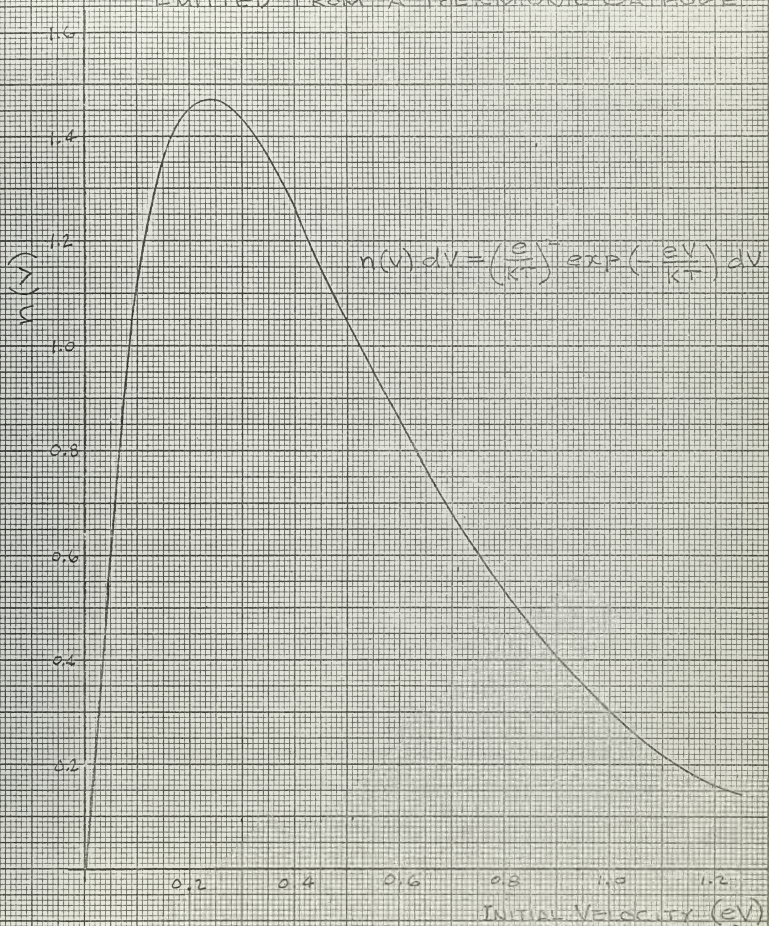
Figure IV-1 shows the distribution of a typical thermionic cathode. Most emitters used in present day electron guns have the peak of this curve in the range of a few tenths of an electron volt. [4,8]

A secondary cause of chromatic aberration is fluctuation of voltages applied to the accelerating electrode. Proper power supply stabilization will make this effect negligible.¹⁷ The Einzel Lens is used to cancel this effect. [7]

Figure IV-2 shows the paths of two electrons at dif-

¹⁷Zworykin et al., op. cit., Sections 6.4 and 16.10.

FIG. IV-1 DISTRIBUTION OF ELECTRON VELOCITIES
EMITTED FROM A THERMIONIC CATHODE



fering velocities through a simple lens. The result is a shift in the focal point from a true paraxial focus to a small circle of confusion beyond the ideal focal point. Chromatic aberration may be measured as the size of this circle (lateral aberration) or in the shift in focus (longitudinal aberration). This study will deal with the for-

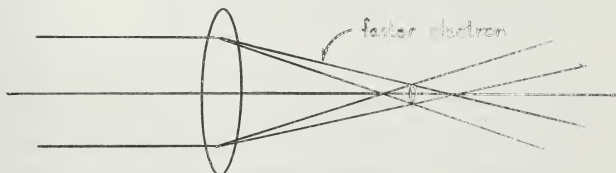


Fig. IV-2 Circle of Least Confusion due to Chromatic Aberration

mer measure. Zworykin states [6] that chromatic aberration depends on the ratio of the initial velocity to the overall voltage and upon the configuration of the lens. We have endeavored to determine the effects of lens configuration as well as check on the direct dependence of the size of the aberration on the ratio of initial velocity to accelerating voltage.

Since chromatic aberration is a first order lens defect, use can be made of the first order solution to Laplace's equation for the trajectories through an electrostatic axially symmetric field. [6,7,8] This solution is

the "paraxial ray" equation. [1,6,7,8]

$$r''(z) + r'(z) \frac{V_0'(z)}{2V_0(z)} + r(z) \frac{V_0''(z)}{4V_0(z)} = 0$$

where: $r(z)$ = radial distance of ray from axis
 z = axial distance
 V_0 = potential measured along the axis
 V_0' = first derivative of axial potential
 V_0'' = second derivative of axial potential

This equation was programmed on the Control Data 1604 Digital Computer such that the lateral and/or longitudinal Chromatic Aberration Coefficient and lens focal properties were computed from an input of the normalized potential along the axis for any lens. Program CHROMAX in its complete form may be found in Appendix II along with explanations as to input and computational methods used.

The first parameter of interest we checked was the dependence of the aberration on the absolute potential on both electrodes. Figure IV-3 shows the aberration plotted for varying voltages for one Immersion Lens. This plot is representative of all lenses studied and is for an initial velocity spread of one electron volt. Results for a spread of ten electron volts were exactly ten times as big for the two cylinder lenses of Systems A and B and slightly less than ten times as big for the three-element lenses. Note that the aberration is precisely inversely propor-

tional to the accelerating voltage (V_1), and is considerably less dependent on the second cylinder voltage. The size of the circle of least confusion may be found from figure IV-3 using the following formula:

$$r = r_a (\Delta\phi) C_{cr}$$

where: r_a = radius of aperture
 $\Delta\phi$ = initial spread of velocities

For example, a lens with diameter = 3 cm., operating at 2000v and 7500v, we may consider an initial velocity spread of 0.3 electron volts. If there is an aperture of 0.1 lens diameter in the system, we get the resulting circle of least confusion to be:

$$r_{cr} = (.3)(.3)(5.4 \times 10^{-5}) = 5 \times 10^{-6} \text{ cm.}$$

The comparison of various configurations is presented in figures IV-4, IV-5, and IV-6. These plots are for a spread of one electron volt initial velocities and for $V_1 = 1000$ volts. The size of the circle of least confusion may be obtained from figures IV-4, IV-5, or IV-6 by the following formula:

$$r_{cr} = r_a \frac{\Delta\phi}{V_1 (\text{kv})} C_{cr}$$

where the parameters are the same as before and V_1 is the accelerating voltage in kilovolts. The parameter $\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^2$ against which the aberration coefficient is plotted is representative of lens strength. Plots include both initially accelerating and initially decelerating fields. Note that in general, all lenses have approximately the same order of magnitude aberration, and initially decelerating fields have considerably higher aberration than the initially accelerating fields. In all cases for the three element lenses, $V_1 = V_3$.

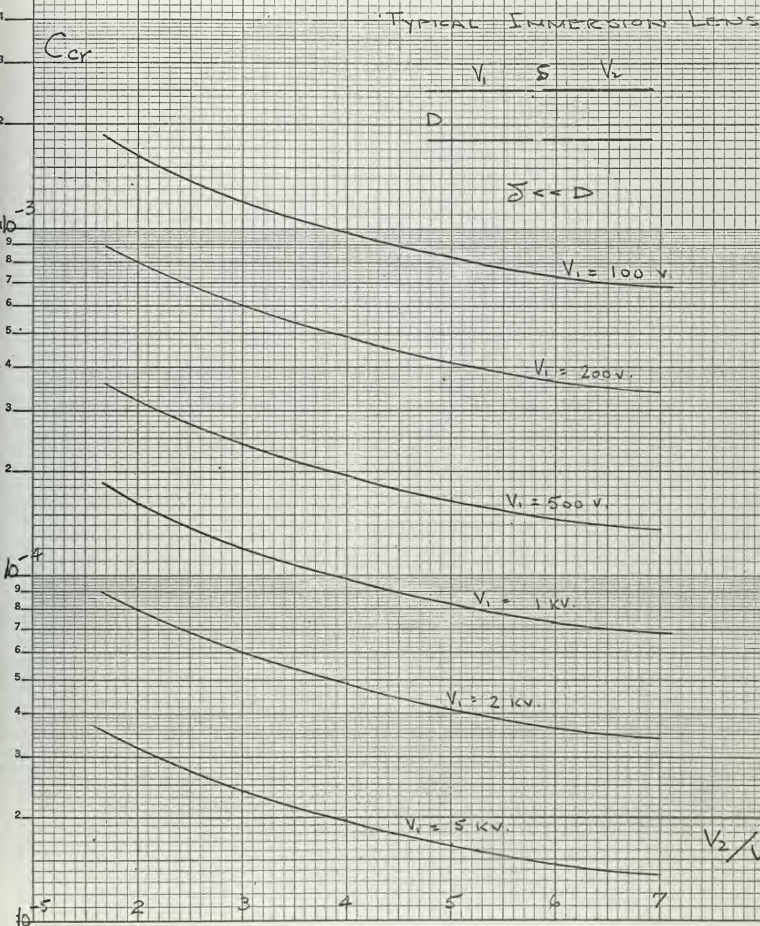
It is interesting to note that the Einzel Lenses are less sensitive to changes in V_2 than the other lenses studied. No lens stands out as being particularly ideal in reducing chromatic aberration, but the following trends are noteworthy. As the voltages on any lens are raised, chromatic aberration becomes negligible and is normally ignored when the accelerating voltage is above a few kilovolts. Increased spacing between electrodes reduces the aberration for all lenses studied. The Immersion Lens with different diameter electrodes has an aberration coefficient twice as large as the lenses with equal diameter. Changes in diameter ratio for the Coaxial Cylinder Lens have negligible effect. Flaring the ends of the electrodes had negligible effect and the results can not be distinguished on the curves as plotted. The effect of the length of the center cylinder in the Einzel Lens is dependent on whether

the first field is accelerating or decelerating. The aberration is considerably less for the case of an initially accelerating field, in which an increase in center cylinder length reduces the aberration. Changes in mid-cylinder length have the opposite effect for initially decelerating fields. The aperture lenses are comparable to the two cylinder lenses, but the spacing of the two main cylinders has little effect on the aberration.

Chromatic aberration can be reduced to negligible limits in electrostatic lenses by using high enough accelerating voltages. One must be careful in going to too high a voltage because relativistic effects begin to occur above 10KV. [6,7,10] Complete elimination of chromatic aberration is not possible in simple electrostatic lenses. Various schemes have been proposed to eliminate it by use of electron mirrors and superimposed electrostatic and magnetic fields. [4,8]

Following the chapter on spherical aberration there will be a comparison of the two aberrations, and the best operating ranges will be discussed for the lenses studied.

FIG. IV-3

VARIATION OF CHROMATIC ABERRATION
WITH ABSOLUTE POTENTIAL

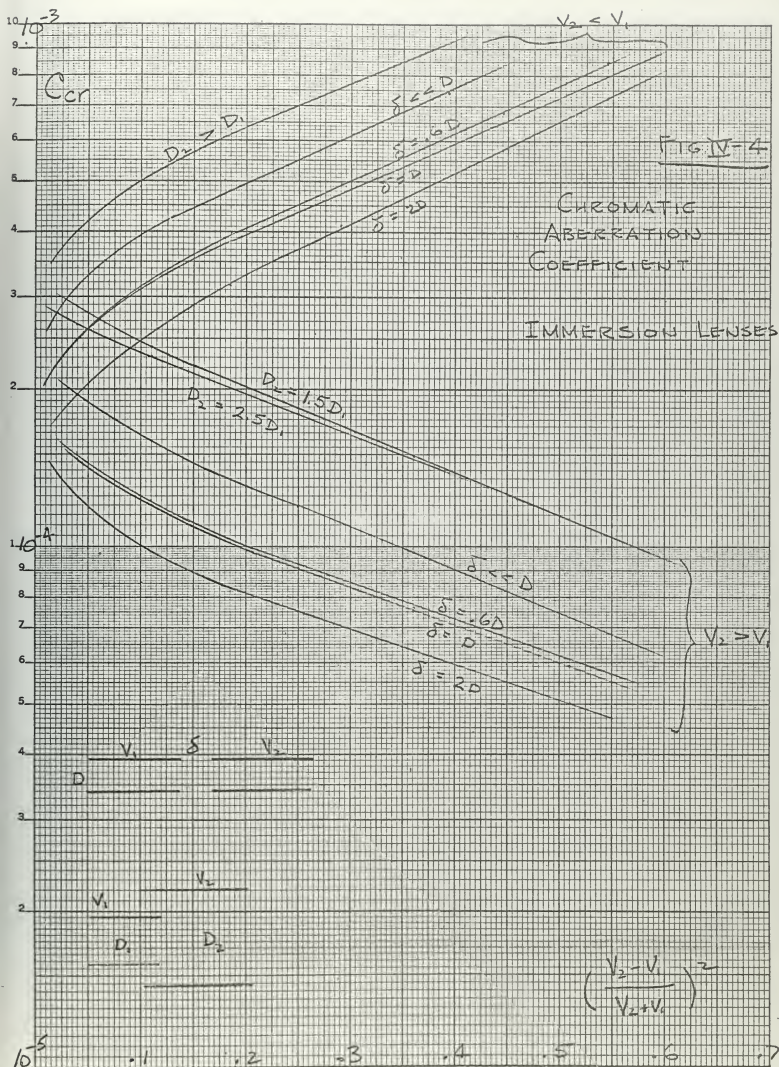


FIG. IV-5 CHROMATIC ABERRATION COEFFICIENT

FINEEL LENSES

$$\frac{V_1}{D} \quad \frac{V_2}{\delta} \quad \frac{V_1}{l}$$

C_{cr}

$\delta = .6 D$

$\delta \ll D$

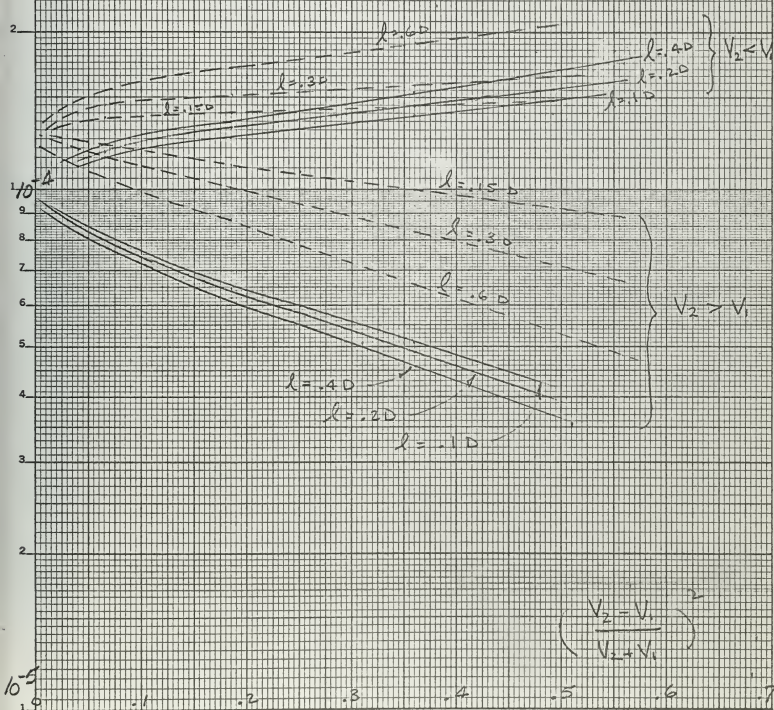
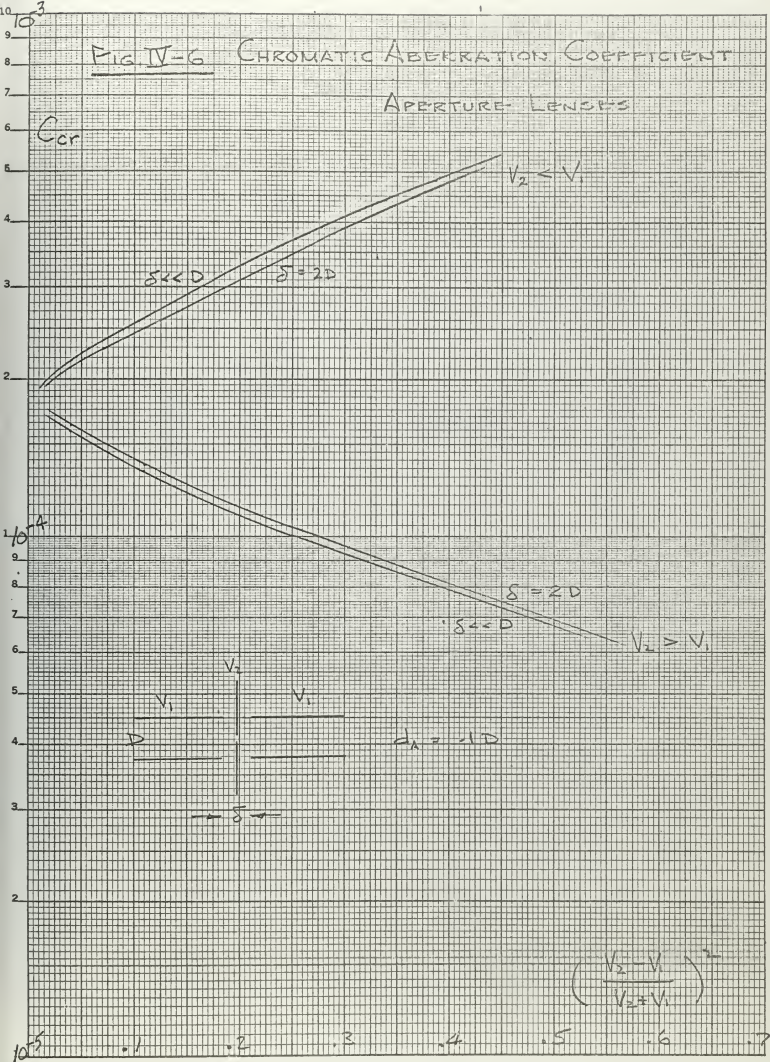


FIG. IV-6 CHROMATIC ABERRATION COEFFICIENT

APERTURE LENSES



CHAPTER V

SPHERICAL ABERRATION

Spherical aberration is due to rays traveling off the axis being bent more than those nearer the axis. Figure V-1 shows the result of this phenomenon. Instead of a perfect focus, a small circle of least confusion appears dis-



Fig. V-1 Cause of Spherical Aberration

placed towards the lens. Note that this is in the opposite direction from the effect of chromatic aberration. Note also in figure V-1 that the relative size of the two circles in object space has changed in image space. Rays passing through object points at increasing angles will traverse the lens at larger distances off the axis and will each undergo stronger amounts of bending. The result is that object points will become small circles of confusion on the screen.¹⁸ The size of this circle is a measure of

¹⁸Zworykin and Morton, op. cit., p. 163.

the spherical aberration of the lens.

To compare the aberrations of many lenses, a great number of parameters must be considered. It is impossible to hold all of them constant for lenses of such differing optical properties as we have analyzed. Besides the varying lens strengths, the geometry of lens location, object to image distance, magnification, aperture size, and aperture location all play an important role in determining the aberration. For example, there are an infinite number of positions where a given lens may be placed to form an image within a particular object to image distance. People interested in electron microscope^[6] work deal with cases of large magnification where the object is near the focal point. In focusing systems of cathode ray tubes, where the lens is nearer the image plane than in the case of the microscope, the objective is to obtain a small spot, i.e., a smaller magnification is desired.

We will deal primarily with cases of small magnification in this study. Zworykin^[4] has derived analytical expressions for all the aberrations. The derivation of the aberration equation is based on third-order aberration theory. This postulates that only rays that deviate small amounts from the axis are considered, so that all terms of higher order may be neglected. Small apertures are usually placed at one or more places in cathode-ray tube devices such that this theory predicts results actually measured

in practice. Zworykin's derivation postulates an aperture to exist at the exit pupil of the lens in "field-free" space.¹⁹ The expression shown below when multiplied by the cube of the aperture radius will give the radius of the circle of confusion resulting from spherical aberration.²⁰

$$S_s = \frac{M}{32 r_{a0}^3 \phi_0^{3/2}} \int_{z_0}^{z_L} \phi^{3/2} dz^4 \left\{ -\frac{3}{2} \left(\frac{\phi''}{\phi} \right)^2 - 4 \frac{\phi' \phi'''}{\phi^2} \left(\frac{r_a'}{r_a} \right) \right. \\ \left. + 4 \frac{\phi''}{\phi} \left[\left(\frac{r_a'}{r_a} \right)^2 + \frac{r_a''}{r_a} \right] \right. \\ \left. + \frac{3}{4} \frac{\phi'^2 \phi''}{\phi^3} + 16 \left(\frac{r_a'}{r_a} \right)^4 \right\} dz$$

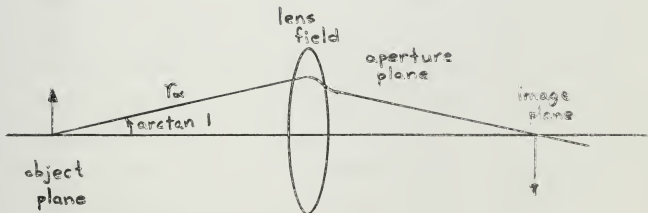


Fig. V-2 Principle Trajectory for Spherical Aberration Coefficient Calculation

¹⁹Zworykin et al., op. cit., see footnote, p. 424.

²⁰Ibid., p. 554, p. 604.

The r_{α} trajectory shown in figure V-2 is a special solution of the paraxial ray equation that vanishes in both the object and image plane and has an initial slope of unity.²¹ $r_{\alpha a}$ is the height of this trajectory in the aperture plane. The computer program has been adjusted to normalize all results in terms of lens diameters, i.e., all lenses have been scaled to the same lens diameter and all distance measurements are in terms of lens diameters. Thus to calculate the size of the circle of confusion it is necessary to multiply the Aberration Coefficient by the cube of the aperture radius, normalized with respect to lens diameter, yielding an answer that is a fraction of a lens diameter.

In the design of cathode ray tubes, the object to image distance is usually fixed by other considerations prior to selection of a focusing system. Thus, in order to get a valid comparison of the many lenses we studied, we held the image to object distance fixed. Newton's lens formula^[7] requires a minimum object to image distance for a given lens strength. A modified version of program SPHERAX was utilized to obtain the curves (figures V-3 to V-7) of minimum object to image distance versus lens strength. From these curves and from a knowledge of actual focusing system dimensions, an object to image dis-

²¹See Appendix III for computational methods used in the computer program. Program SPHERAX was developed from DOLPAC prepared by Litton Industries, San Carlos, California.

tance of 40 lens diameters was chosen to study. As the lens position changed, the required focusing voltage ratio also changed. Due to the wide variations of lens strengths, it soon became apparent that three parameters were intimately tied together in the comparison. Hence two curves for each lens have been plotted for an object to image distance of 40 lens diameters. One curve plots the lens position parameter versus the voltage ratio parameter. The other curve plots the aberration coefficient versus the voltage ratio parameter. Both curves are required to analyze the aberration for a given set of conditions. By referring to the curves plotted for minimum object to image distance versus voltage ratio parameter, it is evident that there is quite a disparity between the operating ranges of the various lenses under study. To observe the effect of a change in object to image distance, results were obtained for distances of 40, 50, and 60 lens diameters. Results plotted in figure V-8 are representative of the trend for all lenses studied. Note that an increase in object to image distance, when measured in lens diameters, is the same as a decrease in tube diameter for a fixed distance. This shows that by increasing cylinder diameter, the aberration may be reduced.

General perusal of the aberration curves shows that the two cylinder lenses are one to two orders of magnitude better than the three cylinder lenses with the exception

of one aperture lens. We believe further investigation of this type of lens might show it to be superior with respect to spherical aberration in the low voltage ratio region. Another general conclusion that stands out is the fact that the aberration is smallest at low voltage ratios, i.e., for weak lenses.

A close look at the Immersion Lens curves show that as spacing between electrodes is increased, the aberration is reduced. The aberration is essentially the same for accelerating and decelerating potentials, although this type of a conclusion can sometimes lead to erroneous results because the lens position curve must also be consulted when comparing lenses. If the comparison is to be made at a fixed position, the voltage ratio parameter will be different for two lenses. Note for example, it might be concluded that the decelerating potential case for the Coaxial Cylinder Lenses shows considerable improvement over the accelerating case, but when the lens location is fixed and one consults both curves, the aberrations are approximately equal. Hence caution must be exercised in drawing general conclusions from any one curve. It is interesting to note that by arbitrary flaring of the ends of the cylinders, the aberration was increased when the tube diameters were equal and was reduced for the case of the coaxial system. We believe that there is probably an optimum flare radius for a given lens configuration that

will reduce the aberration. The flare radius chosen in the latter case fit the equipotential surface of the field more closely than the former. By proper flaring, the optimum lens of Plass is approached.[2]

The Einzel lenses show the same resulting decrease in aberration with increased spacing as the Immersion Lenses. Increasing mid-cylinder length has the same effect. These curves indicate that the range of voltage ratios over which this type of lens can be used is quite narrow for the given object to image distance. Further investigation of other object to image distances might show a flattening of the curves as seen in the Aperture Lens case.

FIG. V-3

VARIATION OF MINIMUM OBJECT - IMAGE DISTANCE

EQUIDIAMETER CYLINDER LENS (IMMERSION)

O-I DIST_(MIN)
(DIAMETERS)

δ
D

a: $\delta/D = 2$

b: $\delta/D = 1$

bf: $\delta/D = 1$ W/FLARE

c: $\delta/D = .6$

cf: $\delta/D = .6$ W/FLARE

d: $\delta/D \ll 1$

$$\left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2$$

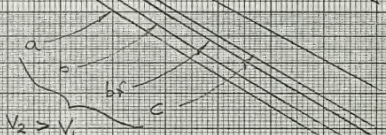
FIG. V-4

VARIATION OF MINIMUM OBJECT-IMAGE DISTANCE

COAXIAL CYLINDER LENS (IMMERSION)

O-I DIST (MIN)
(DIAMETERS)

$V_2 < V_1$

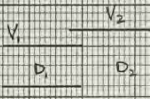


a : $D_2/D_1 = 1.5$

b : $D_2/D_1 = 2.0$

c : $D_2/D_1 = 2.5$

bf : $D_2/D_1 = 2.0$ D_1 ENDS FLARED



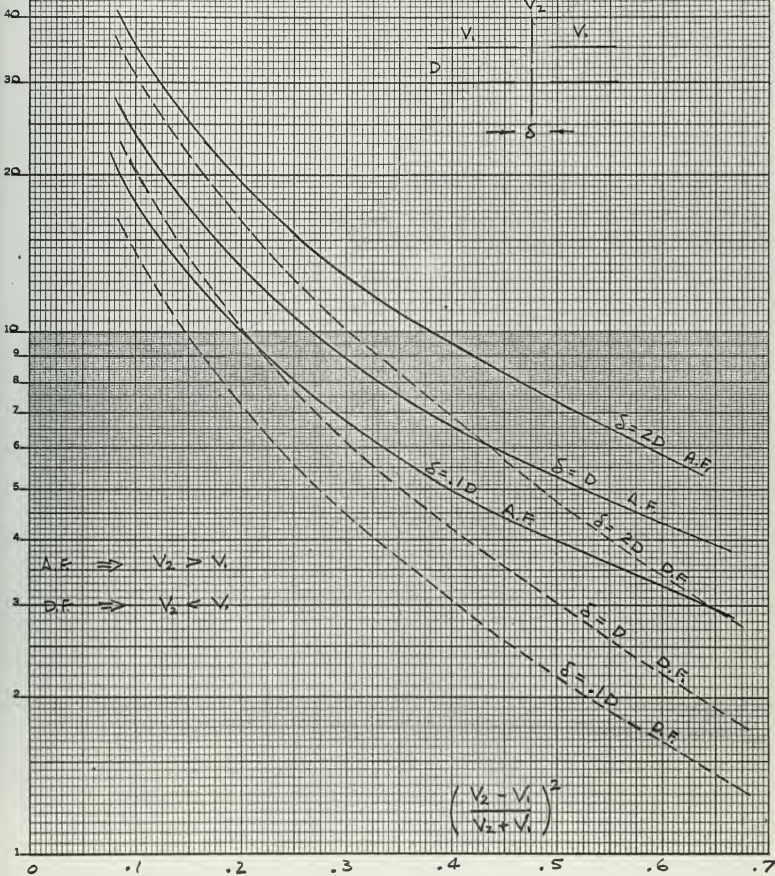
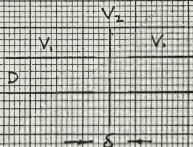
$$\left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2$$

O-I DIST (MIN)
(DIAMETERS)

FIG. V-5

VARIATION OF MINIMUM
OBJECT - IMAGE DISTANCE

APERTURE LENS



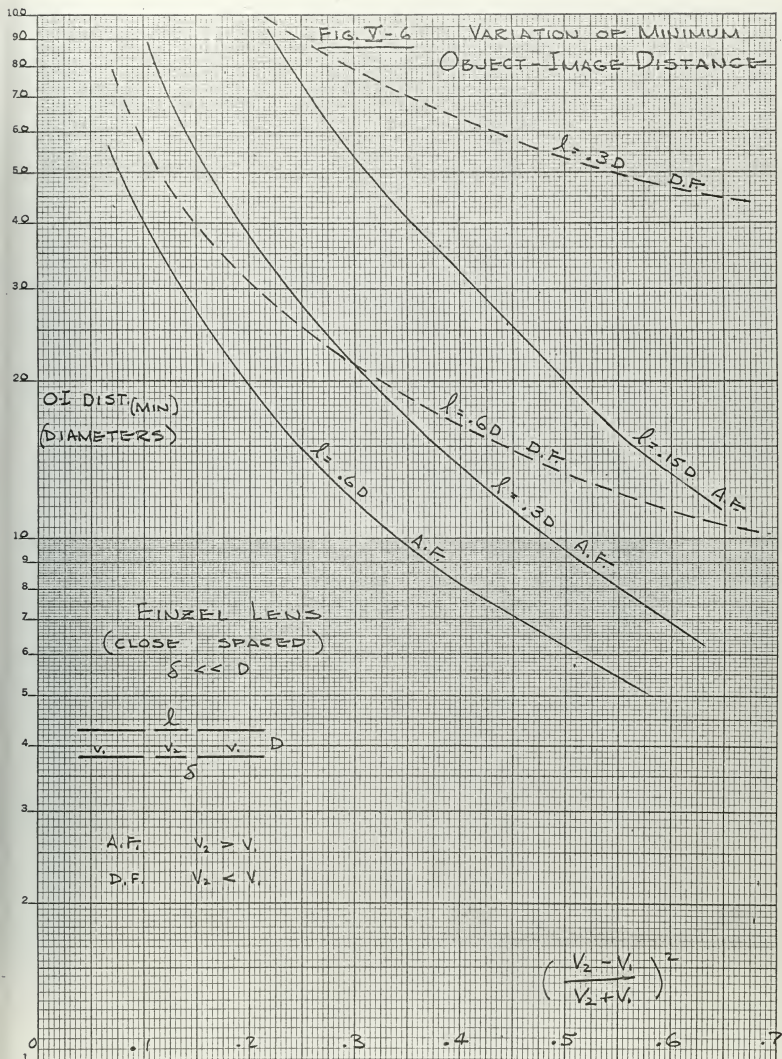


FIG. X-7

VARIATION OF MINIMUM OBJECT-IMAGE DISTANCE

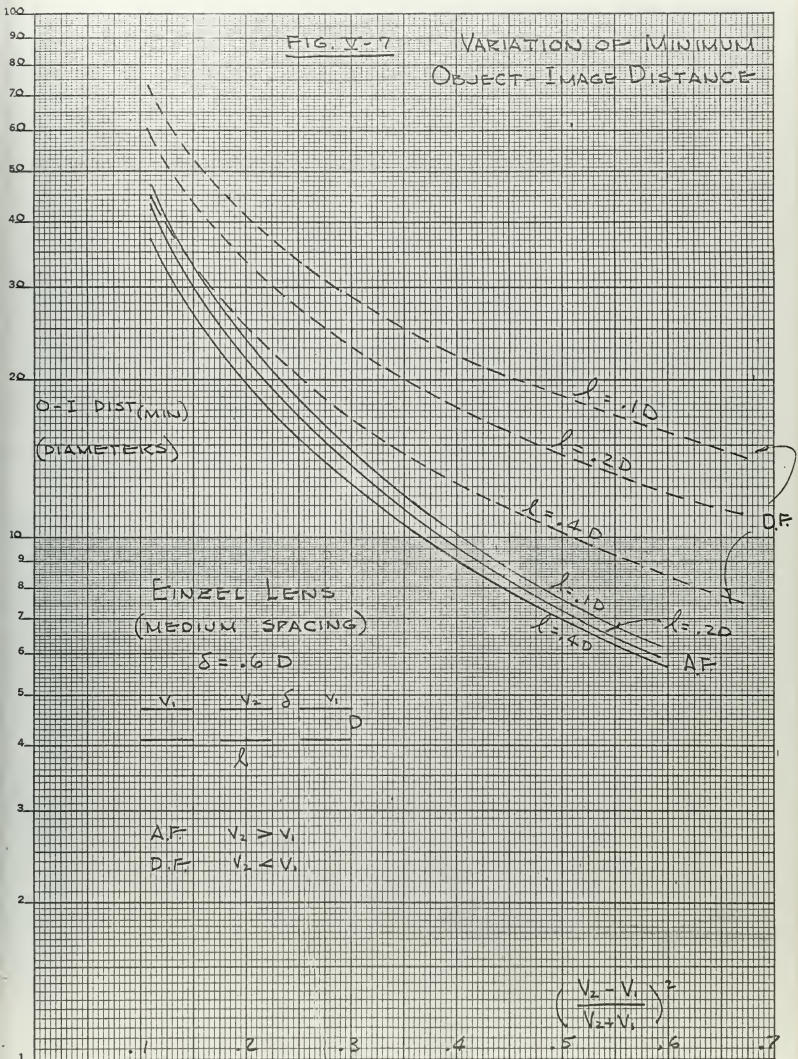


FIG. V-8 VARIATION OF SPHERICAL ABERRATION
COEFFICIENT
WITH
OBJECT TO IMAGE DISTANCE
(DISOI)

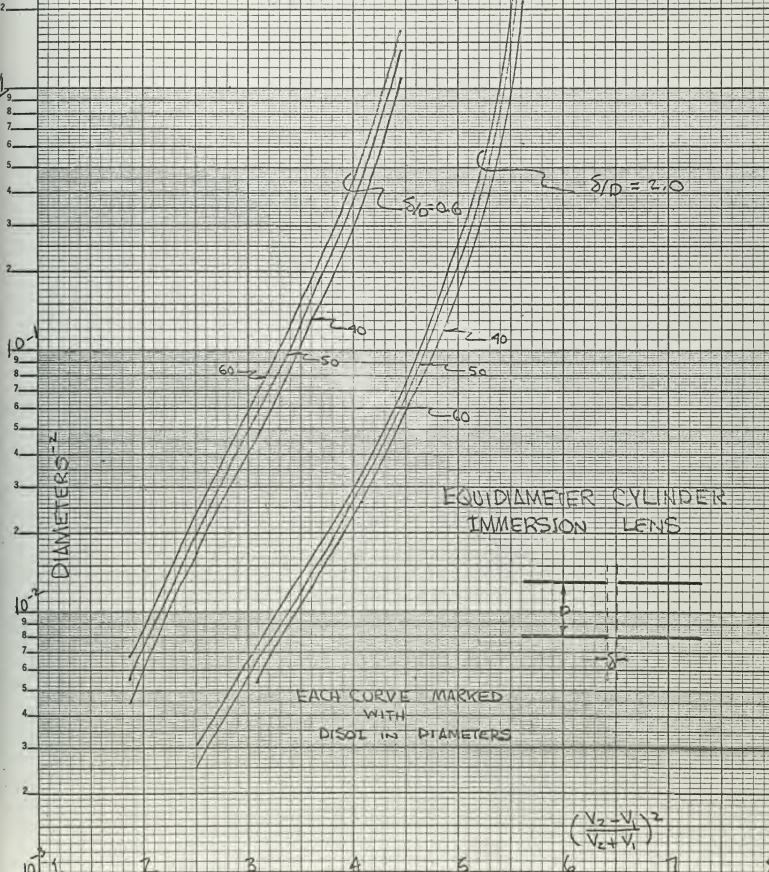


FIG. V-9

SPHERICAL ABERRATION

COEFFICIENT

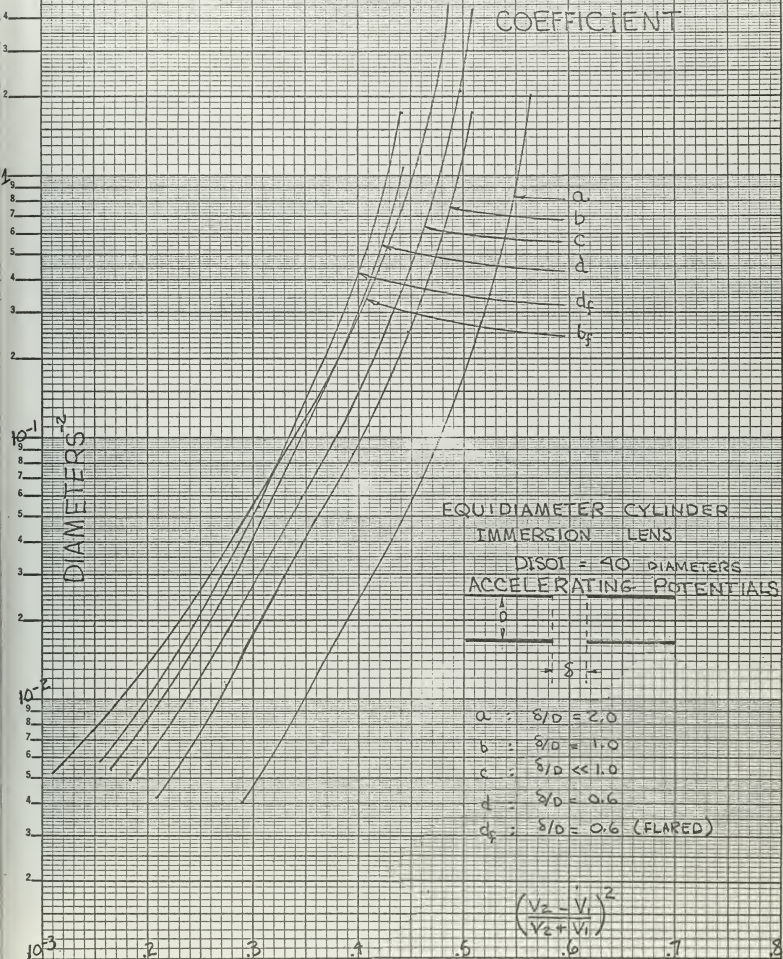
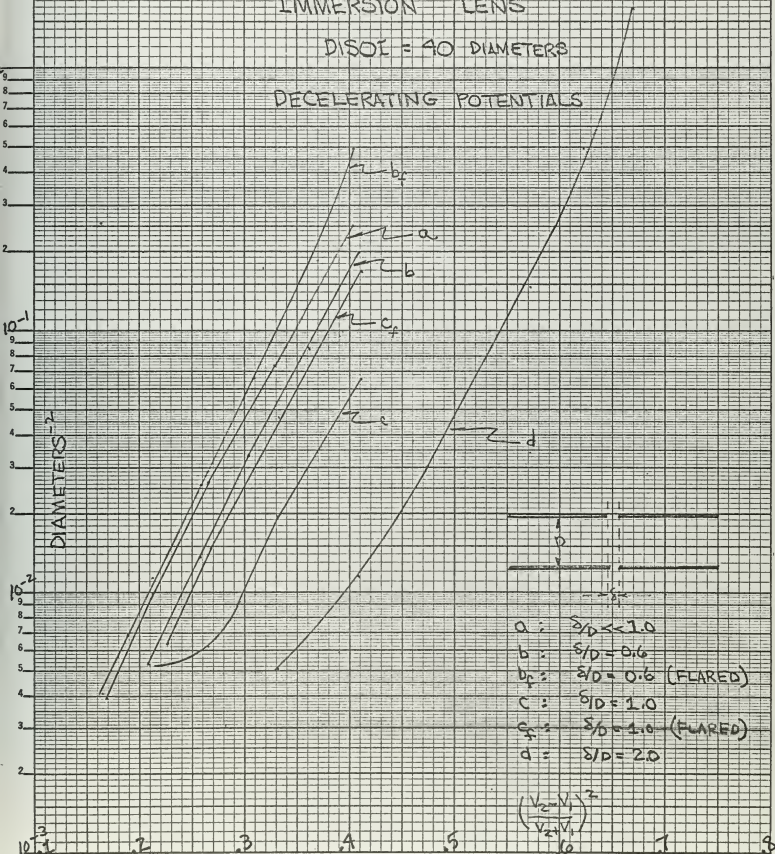


FIG. V-10 SPHERICAL ABERRATION
COEFFICIENT

EQUIDIAMETER CYLINDER
IMMERSION LENS

DISOI = 40 DIAMETERS

DECELERATING POTENTIALS



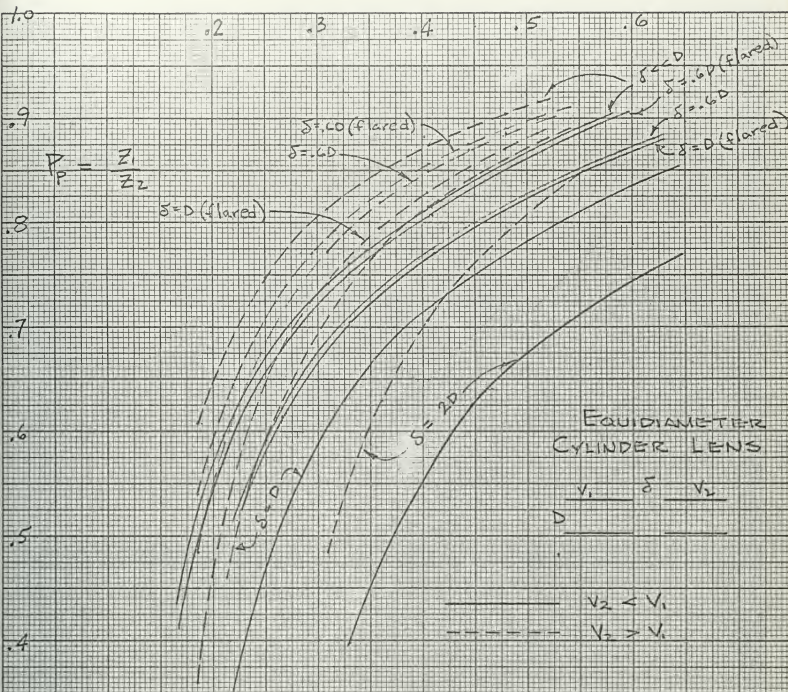
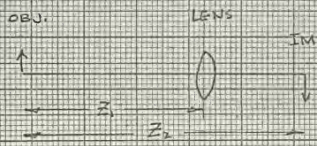


FIG V-11

VARIATION OF LENS POSITION
WITH LENS STRENGTH

OBJECT TO
IMAGE DISTANCE = 40 DIAM.



$$\left(\frac{v_2 - v_1}{v_2 + v_1} \right)^2$$

FIG. Y-12 SPHERICAL ABERRATION COEFFICIENT

COAXIAL CYLINDER
IMMERSION LENS

DISOI = 40 DIAMETERS

ACCELERATING POTENTIALS

a
b
b_f
c



- a : $V/d = 1.5$
- b : $V/d = 2.0$
- b_f : $V/d = 2.0$ (FLARED)
- c : $V/d = 2.5$

$$\left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2$$

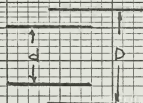
DIAMETERS⁻²

FIG. V-13 SPHERICAL ABERRATION COEFFICIENT

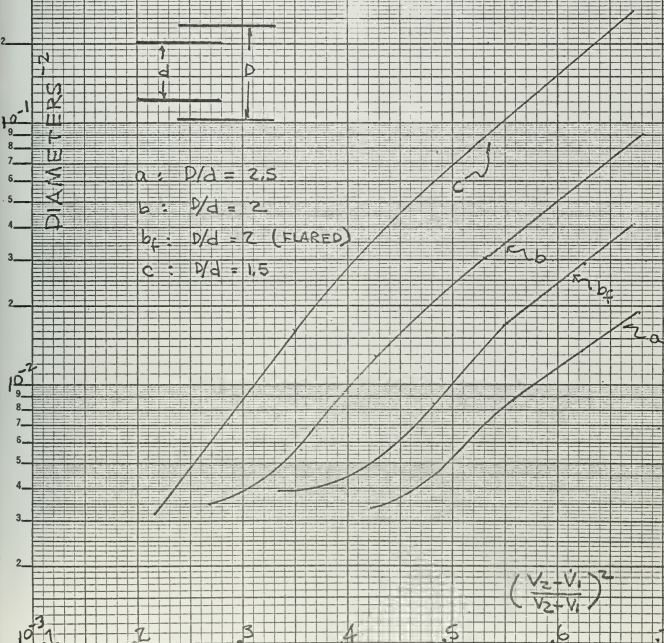
COAXIAL CYLINDER
IMMERSION LENS

DISOI = 40 DIAMETERS

DECELERATING POTENTIALS



- a: $D/d = 2.5$
- b: $D/d = 2$
- b_f: $D/d = 2$ (FLARED)
- c: $D/d = 1.5$



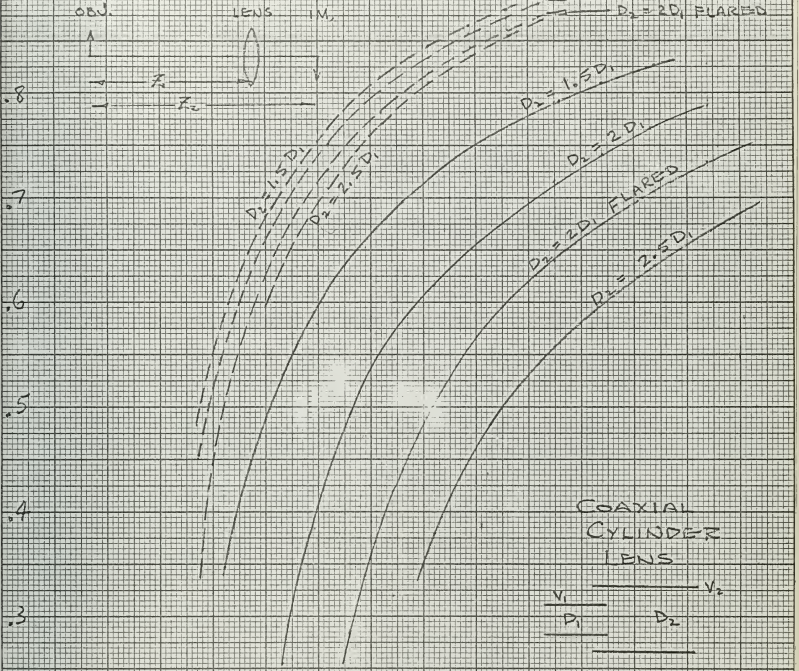
1.0

FIG. V-14

VARIATION OF LENS POSITION WITH LENS STRENGTH

$$P_p = \frac{Z_1}{Z_2}$$

0.9



COAXIAL
CYLINDER
LENS



0.2

————— DECELERATING ($V_2 < V_1$)
 - - - - - ACCELERATING ($V_2 > V_1$)

OBJECT TO IMAGE DISTANCE = $40 D_1$

0.1

$$\left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2$$

0.1 0.2 0.3 0.4 0.5 0.6 0.7

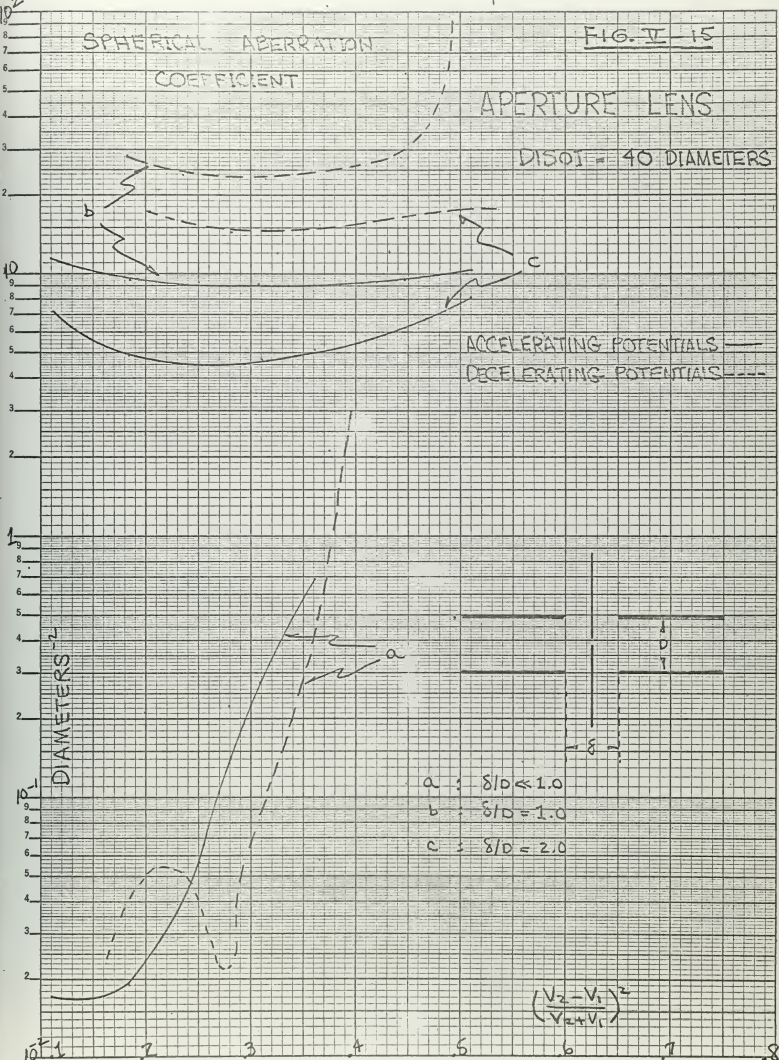
SPHERICAL ABERRATION COEFFICIENT

FIG. II-15

APERTURE LENS

DISOI = 40 DIAMETERS

ACCELERATING POTENTIALS —
DECELERATING POTENTIALS - - -



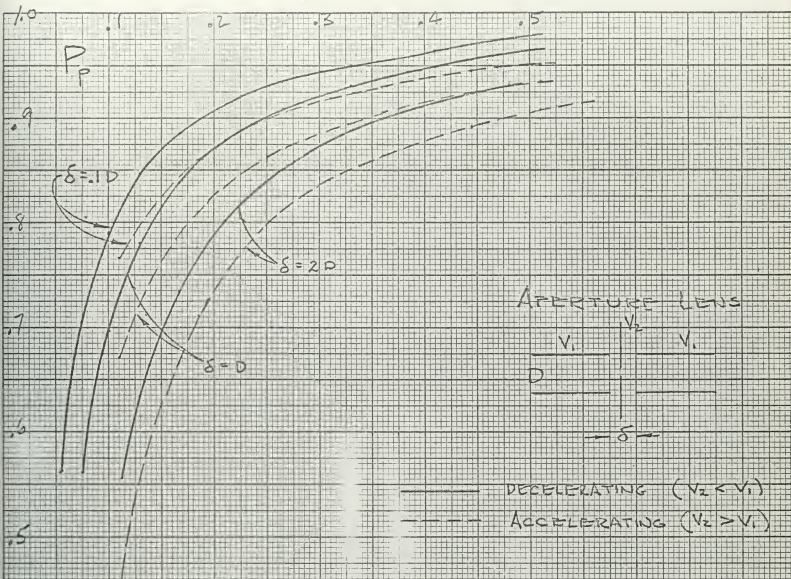


FIG. I-16

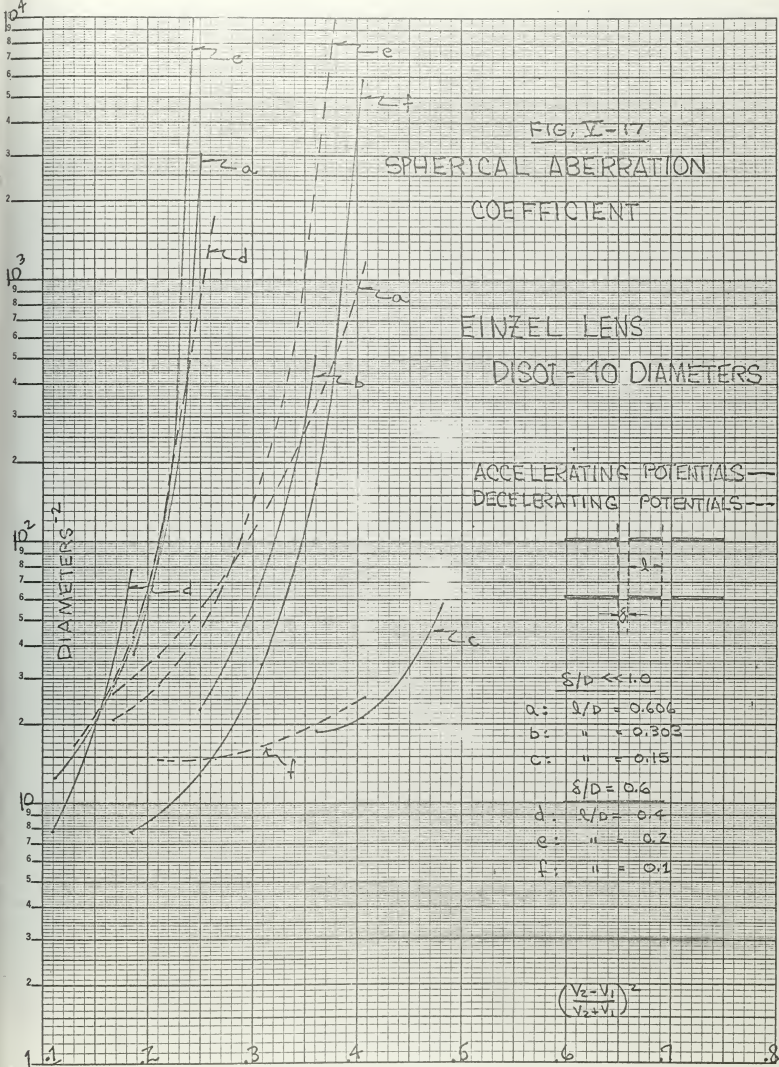
VARIATION OF LENS POSITION
WITH LENS STRENGTH

OBJECT TO IMAGE DISTANCE = 40 DIAMETER

$$P_p = \frac{Z_1}{Z_2}$$



$$\left(\frac{V_2 - V_1}{V_2 + V_1} \right)^2$$



- $s/D < 1.0$
- a: $s/D = 0.606$
 - b: " = 0.302
 - c: " = 0.15
- $s/D = 0.6$
- d: " = 0.4
 - e: " = 0.2
 - f: " = 0.1

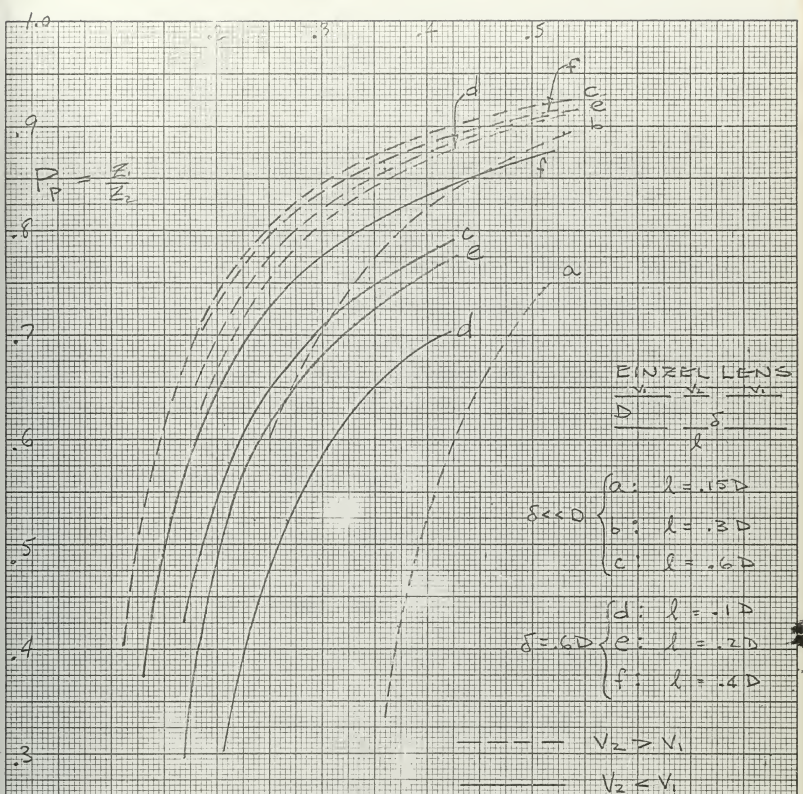
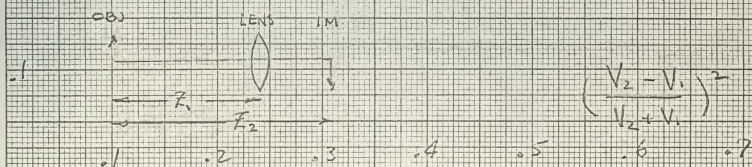


FIG. IV-18

VARIATION OF LENS
POSITION W/ LENS STRENGTH



CHAPTER VI

SUMMARY, CONCLUSIONS, AND ACKNOWLEDGEMENTS

The curves plotted in this paper provide a means of comparing chromatic and spherical aberration given the conditions of operation for the lens. As it can be seen from the curves, these aberrations differ widely in magnitude depending on the lens position, object to image distance, and the operating potentials. A proper choice of operating point can result in minimum aberration.

The choice of physical aperture size is of considerable importance, since both aberrations depend on this limiting factor. Recall that:

$$r_{cr} = r_a \frac{\Delta\phi}{V_i} C_{cr}$$

$$r_s = r_a^3 C_s$$

Also of considerable importance is the fact that the beam current is proportional to the square of the radius of the aperture in accordance with the expression below:

$$i_b = \pi r_a^2 J$$

Thus, as the aperture size is reduced, the beam current is also reduced.

As an example of aberration calculations, let us ex-

amine a coaxial cylinder lens operating under the following conditions, where a fixed position is imposed and voltages can be varied:

$$\begin{aligned}d_1 &= 1 \text{ cm} \\d_2 &= 2 \text{ cm} \\ \text{object-image} &= 40 \text{ cm} \\ \text{lens position} &= 24 \text{ cm beyond crossover} \\ \Delta\phi &= 0.3 \text{ ev}\end{aligned}$$

From figure V-14 we see that we must operate at the following voltage parameters:

$$\begin{aligned}\text{accelerating:} & 0.225 \\ \text{decelerating:} & 0.394\end{aligned}$$

Figures V-12 and V-17 give the spherical aberration coefficients:

$$\begin{aligned}\text{accelerating:} & 6.2 \times 10^{-3} \\ \text{decelerating:} & 9.2 \times 10^{-3}\end{aligned}$$

If an aperture of $d_a = 0.2$ is used, the resulting circles of confusion due to spherical aberration will have radii respectively of:

$$\begin{aligned}\text{accelerating:} & 6.2 \times 10^{-5} \text{ cm} \\ \text{decelerating:} & 9.2 \times 10^{-5} \text{ cm}\end{aligned}$$

From figure IV-4, we get the chromatic aberration coefficient. If 100v is used on the first cylinder, the chromatic aberration will be:

$$\begin{aligned}\text{accelerating:} & 0.57 \times 10^{-5} \text{ cm} \\ \text{decelerating:} & 2.8 \times 10^{-5} \text{ cm}\end{aligned}$$

Note that in the case of accelerating potentials chromatic aberration is a factor of ten smaller than spherical and can be neglected, but in the decelerating case, chromatic aberration is only three times smaller and cannot be neglected.

Now consider a case where an Equidiameter Cylinder Immersion Lens is being operated at a fixed 4.5 to 1 voltage ratio, and position can be varied. We will choose the widely spaced lens as an example:

diameter = 1 cm
 voltage parameter = 0.405
 accelerating position = 28.2 cm beyond crossover
 decelerating position = 22.5 cm beyond crossover
 accelerating sph. ab. coef. = 2.9×10^{-2}
 decelerating sph. ab. coef. = 1.2×10^{-2}

Again considering an aperture of 0.2 cm, we find the resulting spherical aberration to be:

accelerating: $r_s = 2.9 \times 10^{-5} \text{ cm}$
 decelerating: $r_s = 1.2 \times 10^{-5} \text{ cm}$

The chromatic aberration when 100 volts is applied to the first cylinder under the same conditions will be:

accelerating: $r_{cr} = 1.8 \times 10^{-5} \text{ cm}$
 decelerating: $r_{cr} = 1.6 \times 10^{-4} \text{ cm}$

Here we find that operation at low potential is limited by chromatic aberration and not spherical as in the first example. We must therefore go to higher potentials for the

given conditions to reduce chromatic aberration.

It is evident that there is no simple relation between spherical and chromatic aberration for any lens because of the multiplicity of parameters involved. Determination of the limiting aberration must be calculated from the curves for each given situation. It appears that best operating points occur for weak lenses near the minimum object to image distance. We believe that the curves derived in this study provide a good foundation for determining an optimum lens for a given application. Digital computer analysis is an excellent tool in this field and there is obviously much that computers can contribute in the area of lens design. It is recommended that as a topic for further research in this field, a computer study aimed at optimizing the object to image distance for various lenses be made.

The writers wish to express their appreciation to members of the faculty and the Postgraduate School Computer Facility. We are especially grateful to Dr. Glenn A. Gray, our faculty advisor, who made us aware of the need for research in this field and who helped us establish relations with Dr. J. Pokorny and Mr. G. Petersen of the Display Devices Section of the Electron Tube Division of Litton Industries. The Litton engineers made available Dr. J. R. Hechtel's resistor analog network for obtaining potential data and gave us considerable insight into the

practical problems involved. Dr. J. R. Ward, Professor of Electrical Engineering, United States Naval Postgraduate School, was extremely helpful in providing his numerical integration scheme for solving differential equations. Finally, we would be amiss if we did not express appreciation for the patience and understanding of our wives who at times must have been convinced that they had been superseded by a couple of high-speed, solid state, expensive machines, the IBM 1401 and CDC 1604.

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APPENDIX I

SMOOTHED NORMALIZED POTENTIAL DATA FOR THE LENSES STUDIED

The digital voltmeter readings, after being smoothed out using the technique shown in figure II-1, are listed here for the benefit of others who might wish to pursue this subject further.

IMMERSION LENS
SPACING TO DIAMETER RATIO =2.0

Z	V(NORM)	Z	V(NORM)
1	.0000000	51	.5000000
2	.0000000	52	.5237000
3	.0000000	53	.5474000
4	.0000137	54	.5713000
5	.0000183	55	.5954000
6	.0000245	56	.6199000
7	.0000330	57	.6447000
8	.0000440	58	.6701000
9	.0000590	59	.6959000
10	.0000810	60	.7223000
11	.0001070	61	.7491000
12	.0001440	62	.7763000
13	.0001920	63	.8035000
14	.0002550	64	.8304000
15	.0003400	65	.8566000
16	.0004600	66	.8812000
17	.0006200	67	.9035000
18	.0008400	68	.9232000
19	.0012000	69	.9399000
20	.0015000	70	.9535000
21	.0020000	71	.9645000
22	.0027000	72	.9730000
23	.0036000	73	.9795000
24	.0048000	74	.9847000
25	.0064000	75	.9885000
26	.0086000	76	.9914000
27	.0115000	77	.9936000
28	.0153000	78	.9945000
29	.0204000	79	.9964000
30	.0270000	80	.9973000
31	.0355000	81	.9980000
32	.0465000	82	.9985000
33	.0601000	83	.9988000
34	.0769000	84	.9991600
35	.0965000	85	.9993800
36	.1188000	86	.9995400
37	.1434000	87	.9996600
38	.1696000	88	.9997450
39	.1965000	89	.9998070
40	.2237000	90	.9998560
41	.2509000	91	.9998930
42	.2777000	92	.9999200
43	.3041000	93	.9999410
44	.3299000	94	.9999560
45	.3553000	95	.9999670
46	.3801000	96	.9999755
47	.4046000	97	.9999817
48	.4287000	98	.9999863
49	.4526000	99	1.0000000
50	.4763000	100	1.0000000
		101	1.0000000

IMMERSION LENS
SPACING TO DIAMETER RATIO = 1.0

Z	V(NORM)	Z	V(NORM)
1	.0000000	51	.5000000
2	.0000020	52	.5374000
3	.0000040	53	.5747000
4	.0000060	54	.6118000
5	.0000080	55	.6486000
6	.0000100	56	.6850000
7	.0000120	57	.7206000
8	.0000140	58	.7550000
9	.0000160	59	.7878000
10	.0000180	60	.8186000
11	.0000200	61	.8469000
12	.0000220	62	.8723000
13	.0000240	63	.8946000
14	.0000260	64	.9138000
15	.0000280	65	.9301000
16	.0000300	66	.9437000
17	.0000320	67	.9548000
18	.0000340	68	.9640000
19	.0000360	69	.9713000
20	.0000380	70	.9772000
21	.0000400	71	.9820000
22	.0000420	72	.9857000
23	.0000440	73	.9887000
24	.0000460	74	.9911000
25	.0000480	75	.9929000
26	.0000500	76	.9946000
27	.0000520	77	.9956000
28	.0000540	78	.9965000
29	.0000560	79	.9972000
30	.0000580	80	.9978000
31	.0000600	81	.9982000
32	.0000620	82	.9986000
33	.0000640	83	.9989000
34	.0000660	84	.9991200
35	.0000680	85	.9993000
36	.0000700	86	.9994500
37	.0000720	87	.9995600
38	.0000740	88	.9996500
39	.0000760	89	.9997250
40	.0000780	90	.9997800
41	.0000800	91	.9998170
42	.0000820	92	.9998620
43	.0000840	93	.9998900
44	.0000860	94	.9999120
45	.0000880	95	.9999300
46	.0000900	96	.9999490
47	.0000920	97	.9999555
48	.0000940	98	.9999650
49	.0000960	99	.9999720
50	.0000980	100	.9999780
		101	1.0000000

IMMERSION LENS (FLARED ENDS)
SPACING TO DIAMETER RATIO = 1.0

Z	V(NORM)	Z	V(NORM)
1	.0000000	51	.5000000
2	.0000180	52	.5410000
3	.0000230	53	.5800000
4	.0000290	54	.6184000
5	.0000370	55	.6561000
6	.0000470	56	.6931000
7	.0000600	57	.7296000
8	.0000760	58	.7627000
9	.0000960	59	.7948000
10	.0001200	60	.8246000
11	.0001500	61	.8519000
12	.0001900	62	.8763000
13	.0002450	63	.8977000
14	.0003100	64	.9162000
15	.0004000	65	.9320000
16	.0005000	66	.9451000
17	.0006300	67	.9560000
18	.0008000	68	.9648000
19	.0010000	69	.9720000
20	.0013000	70	.9778000
21	.0016800	71	.9824000
22	.0021000	72	.9861000
23	.0026900	73	.9890000
24	.0034000	74	.9913000
25	.0043000	75	.9931000
26	.0054000	76	.9946000
27	.0069000	77	.9957000
28	.0087000	78	.9966000
29	.0110000	79	.9973100
30	.0139000	80	.9979000
31	.0176000	81	.9983200
32	.0222000	82	.9987000
33	.0280000	83	.9990000
34	.0352000	84	.9992000
35	.0440000	85	.9993700
36	.0549000	86	.9995000
37	.0689000	87	.9996000
38	.0838000	88	.9996900
39	.1023000	89	.9997550
40	.1237000	90	.9998100
41	.1481000	91	.9998500
42	.1754000	92	.9998800
43	.2052000	93	.9999040
44	.2373000	94	.9999240
45	.2714000	95	.9999400
46	.3069000	96	.9999530
47	.3439000	97	.9999630
48	.3816000	98	.9999710
49	.4200000	99	.9999770
50	.4590000	100	.9999820
		101	1.0000000

IMMERSION LENS
SPACING TO DIAMETER RATIO = 0.6

Z	V(NORM)	Z	V(NORM)
1	.0000000	51	.5000000
2	.0000119	52	.5515000
3	.0000150	53	.6022000
4	.0000189	54	.6515000
5	.0000239	55	.6985000
6	.0000298	56	.7426000
7	.0000378	57	.7829000
8	.0000475	58	.8190000
9	.0000600	59	.8507000
10	.0000755	60	.8780000
11	.0000950	61	.9011000
12	.0001210	62	.9203000
13	.0001540	63	.9361000
14	.0001900	64	.9490000
15	.0002400	65	.9595000
16	.0003000	66	.9679000
17	.0003900	67	.9746000
18	.0005000	68	.9799000
19	.0006200	69	.9841000
20	.0008000	70	.9874000
21	.0010000	71	.9901000
22	.0012200	72	.9921000
23	.0015000	73	.9938000
24	.0019000	74	.9951000
25	.0024000	75	.9961000
26	.0031000	76	.9969000
27	.0039000	77	.9976000
28	.0049000	78	.9981000
29	.0062000	79	.9985000
30	.0079000	80	.9987800
31	.0099000	81	.9990000
32	.0126000	82	.9992000
33	.0159000	83	.9993800
34	.0201000	84	.9995000
35	.0254000	85	.9996100
36	.0321000	86	.9997000
37	.0405000	87	.9997600
38	.0510000	88	.9998100
39	.0639000	89	.9998460
40	.0797000	90	.9998790
41	.0989000	91	.9999050
42	.1220000	92	.9999245
43	.1493000	93	.9999400
44	.1810000	94	.9999525
45	.2171000	95	.9999622
46	.2574000	96	.9999702
47	.3015000	97	.9999761
48	.3485000	98	.9999811
49	.3979000	99	.9999850
50	.4485000	100	.9999881
		101	1.0000000

IMMERSION LENS (FLARED ENDS)
SPACING TO DIAMETER RATIO = 0.6

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.50000000
2	.00000105	52	.55300000
3	.00000134	53	.60330000
4	.00000170	54	.65200000
5	.00000215	55	.69830000
6	.00000270	56	.74170000
7	.00000340	57	.78140000
8	.00000430	58	.81930000
9	.00000550	59	.84880000
10	.00000695	60	.87610000
11	.00000875	61	.90030000
12	.00001100	62	.91870000
13	.00001400	63	.93420000
14	.00001750	64	.94790000
15	.00002230	65	.95850000
16	.00002850	66	.96700000
17	.00003600	67	.97390000
18	.00004500	68	.97930000
19	.00005800	69	.98370000
20	.00007400	70	.98710000
21	.00009300	71	.98980000
22	.00120000	72	.99200000
23	.00150000	73	.99370000
24	.00190000	74	.99500000
25	.00240000	75	.99610000
26	.00310000	76	.99690000
27	.00390000	77	.99760000
28	.00500000	78	.99810000
29	.00630000	79	.99850000
30	.00800000	80	.99880000
31	.01020000	81	.99907000
32	.01290000	82	.99926000
33	.01630000	83	.99942000
34	.02070000	84	.99954000
35	.02610000	85	.99964000
36	.03300000	86	.99971500
37	.04150000	87	.99977700
38	.05210000	88	.99982500
39	.06520000	89	.99986000
40	.08130000	90	.99989000
41	.10070000	91	.99991250
42	.12390000	92	.99993050
43	.15120000	93	.99994500
44	.18270000	94	.99995700
45	.21860000	95	.99996600
46	.25830000	96	.99997300
47	.30170000	97	.99997850
48	.34800000	98	.99998300
49	.39670000	99	.99998660
50	.44700000	100	.99998950
		101	1.00000000

IMMERSION LENS
SPACING TO DIAMETER RATIO = MUCH LESS THAN 1.0

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.50000000
2	.00005330	52	.54980000
3	.00006550	53	.59870000
4	.00008000	54	.64570000
5	.00009800	55	.69000000
6	.00012000	56	.73110000
7	.00014600	57	.76850000
8	.00018000	58	.80220000
9	.00021900	59	.83200000
10	.00026600	60	.85820000
11	.00032000	61	.88080000
12	.00040000	62	.90020000
13	.00049000	63	.91680000
14	.00060000	64	.93090000
15	.00073000	65	.94270000
16	.00090000	66	.95260000
17	.00110000	67	.96090000
18	.00134000	68	.96770000
19	.00163000	69	.97340000
20	.00200000	70	.97810000
21	.00245000	71	.98200000
22	.00300000	72	.98520000
23	.00370000	73	.98790000
24	.00450000	74	.99000000
25	.00550000	75	.99180000
26	.00670000	76	.99330000
27	.00820000	77	.99450000
28	.01000000	78	.99550000
29	.01210000	79	.99630000
30	.01480000	80	.99700000
31	.01800000	81	.99755000
32	.02190000	82	.99800000
33	.02660000	83	.99837000
34	.03230000	84	.99866000
35	.03920000	85	.99890000
36	.04740000	86	.99910000
37	.05730000	87	.99927000
38	.06910000	88	.99940000
39	.08320000	89	.99951000
40	.09980000	90	.99960000
41	.11920000	91	.99967200
42	.14280000	92	.99973400
43	.16800000	93	.99978100
44	.19780000	94	.99982000
45	.23150000	95	.99985400
46	.26890000	96	.99988000
47	.31000000	97	.99990200
48	.35430000	98	.99992000
49	.40130000	99	.99993500
50	.45020000	100	.99994700
		101	1.00000000

IMMERSION LENS (OVERLAPPED)
DIAMETER RATIO = 1.5

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.31000000
2	.00130000	52	.33500000
3	.00145000	53	.36100000
4	.00164000	54	.38500000
5	.00183000	55	.41500000
6	.00200000	56	.44200000
7	.00230000	57	.47000000
8	.00260000	58	.49900000
9	.00294000	59	.52400000
10	.00330000	60	.55000000
11	.00370000	61	.57600000
12	.00415000	62	.60100000
13	.00460000	63	.62500000
14	.00525000	64	.65000000
15	.00590000	65	.67000000
16	.00665000	66	.69000000
17	.00750000	67	.71100000
18	.00840000	68	.73100000
19	.00940000	69	.75000000
20	.01060000	70	.76700000
21	.01200000	71	.78100000
22	.01340000	72	.79700000
23	.01500000	73	.81000000
24	.01680000	74	.82300000
25	.01900000	75	.83500000
26	.02120000	76	.84600000
27	.02400000	77	.85700000
28	.02690000	78	.86700000
29	.03000000	79	.87500000
30	.03320000	80	.88300000
31	.03800000	81	.89200000
32	.04250000	82	.89900000
33	.04800000	83	.90540000
34	.05330000	84	.91150000
35	.06000000	85	.91700000
36	.06700000	86	.92200000
37	.07600000	87	.92600000
38	.08500000	88	.93050000
39	.09500000	89	.93400000
40	.10700000	90	.93720000
41	.11900000	91	.94000000
42	.13250000	92	.94250000
43	.14600000	93	.94500000
44	.16300000	94	.94740000
45	.17900000	95	.94980000
46	.19700000	96	.95200000
47	.21700000	97	.95400000
48	.23700000	98	.95600000
49	.26100000	99	.95800000
50	.28200000	100	.95980000
		101	.96120000

IMMERSION LENS (OVERLAPPED)
DIAMETER RATIO = 2.0

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.25900000
2	.00100000	52	.28000000
3	.00112000	53	.30300000
4	.00126000	54	.32500000
5	.00142000	55	.35000000
6	.00160000	56	.37000000
7	.00180000	57	.39300000
8	.00202000	58	.42000000
9	.00227000	59	.44600000
10	.00256000	60	.47000000
11	.00289000	61	.49300000
12	.00323000	62	.51500000
13	.00364000	63	.53900000
14	.00410000	64	.56000000
15	.00460000	65	.58100000
16	.00520000	66	.60000000
17	.00585000	67	.62100000
18	.00660000	68	.64000000
19	.00740000	69	.65700000
20	.00836000	70	.67700000
21	.00940000	71	.69100000
22	.01060000	72	.73900000
23	.01200000	73	.72100000
24	.01350000	74	.73700000
25	.01500000	75	.74800000
26	.01700000	76	.76200000
27	.01900000	77	.77200000
28	.02150000	78	.78400000
29	.02420000	79	.79300000
30	.02720000	80	.80400000
31	.03080000	81	.81200000
32	.03450000	82	.82200000
33	.03880000	83	.82900000
34	.04370000	84	.83370000
35	.04920000	85	.84300000
36	.05550000	86	.85000000
37	.06200000	87	.85370000
38	.07000000	88	.86300000
39	.07900000	89	.86900000
40	.08850000	90	.87450000
41	.09800000	91	.88000000
42	.10900000	92	.88500000
43	.12100000	93	.89000000
44	.13450000	94	.89300000
45	.14900000	95	.89800000
46	.16400000	96	.90200000
47	.18100000	97	.90550000
48	.19900000	98	.90900000
49	.21700000	99	.91300000
50	.22700000	100	.91600000
		101	.92000000

IMMERSION LENS (OVERLAPPED AND FLARED)
DIAMETER RATIO = 2.0

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.23600000
2	.00100000	52	.25300000
3	.00115000	53	.27500000
4	.00130000	54	.29500000
5	.00145000	55	.31700000
6	.00160000	56	.33900000
7	.00180000	57	.36100000
8	.00200000	58	.38200000
9	.00220000	59	.40500000
10	.00240000	60	.42800000
11	.00260000	61	.44900000
12	.00290000	62	.47200000
13	.00330000	63	.49300000
14	.00370000	64	.51500000
15	.00410000	65	.53400000
16	.00460000	66	.55500000
17	.00520000	67	.57400000
18	.00580000	68	.59300000
19	.00650000	69	.61100000
20	.00730000	70	.62800000
21	.00820000	71	.64600000
22	.00920000	72	.66200000
23	.01030000	73	.67900000
24	.01150000	74	.69200000
25	.01290000	75	.70700000
26	.01430000	76	.72100000
27	.01630000	77	.73400000
28	.01830000	78	.74600000
29	.02050000	79	.75700000
30	.02300000	80	.76800000
31	.02590000	81	.77800000
32	.02900000	82	.78800000
33	.03250000	83	.79700000
34	.03640000	84	.80550000
35	.04100000	85	.81400000
36	.04600000	86	.82300000
37	.05150000	87	.83000000
38	.05800000	88	.83800000
39	.06550000	89	.84500000
40	.07300000	90	.85200000
41	.08200000	91	.85800000
42	.09100000	92	.86400000
43	.10100000	93	.87000000
44	.11200000	94	.87600000
45	.12500000	95	.88200000
46	.13700000	96	.88700000
47	.15100000	97	.89200000
48	.16600000	98	.89600000
49	.18300000	99	.90100000
50	.19900000	100	.90500000
	.21800000	101	.90900000

IMMERSION LENS (OVERLAPPED)
DIAMETER RATIO = 2.5

Z	V(NORM)	Z	V(NORM)
1	.00000000	51	.22700000
2	.00009000	52	.24500000
3	.00100000	53	.26600000
4	.00120000	54	.28700000
5	.00131000	55	.30900000
6	.00142000	56	.33000000
7	.00160000	57	.35100000
8	.00180000	58	.37200000
9	.00200000	59	.39500000
10	.00225000	60	.41500000
11	.00250000	61	.43700000
12	.00285000	62	.45900000
13	.00320000	63	.47900000
14	.00360000	64	.50000000
15	.00405000	65	.51900000
16	.00456000	66	.52700000
17	.00515000	67	.55500000
18	.00580000	68	.57100000
19	.00650000	69	.59000000
20	.00730000	70	.60600000
21	.00820000	71	.62200000
22	.00930000	72	.63500000
23	.01040000	73	.65100000
24	.01170000	74	.66400000
25	.01320000	75	.67700000
26	.01490000	76	.69000000
27	.01670000	77	.70100000
28	.01880000	78	.71200000
29	.02110000	79	.72200000
30	.02400000	80	.73100000
31	.02700000	81	.74100000
32	.03030000	82	.75000000
33	.03400000	83	.75800000
34	.03850000	84	.76700000
35	.04350000	85	.77300000
36	.04900000	86	.78000000
37	.05500000	87	.78500000
38	.06100000	88	.79000000
39	.06900000	89	.79600000
40	.07650000	90	.80000000
41	.08500000	91	.80500000
42	.09550000	92	.80800000
43	.10600000	93	.81200000
44	.11700000	94	.81500000
45	.13000000	95	.81800000
46	.14450000	96	.82200000
47	.15900000	97	.82500000
48	.17500000	98	.82900000
49	.19100000	99	.83200000
50	.20800000	100	.83500000
		101	.83800000

APERTURE LENS
SPACING TO DIAMETER RATIO = 0.1

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.0470000
2	1.0000000	52	.1410000
3	1.0000000	53	.2610000
4	1.0000000	54	.3780000
5	1.0000000	55	.4830000
6	.9999000	56	.5750000
7	.9998000	57	.6540000
8	.9997000	58	.7200000
9	.9996000	59	.7740000
10	.9995000	60	.8190000
11	.9994000	61	.8550000
12	.9993000	62	.8840000
13	.9992000	63	.9070000
14	.9991000	64	.9250000
15	.9990000	65	.9400000
16	.9989000	66	.9520000
17	.9988000	67	.9615000
18	.9987000	68	.9690000
19	.9986000	69	.9752000
20	.9985000	70	.9802000
21	.9984000	71	.9841000
22	.9983000	72	.9872000
23	.9982000	73	.9898000
24	.9981000	74	.9917000
25	.9980000	75	.9934000
26	.9979000	76	.9947000
27	.9978000	77	.9957400
28	.9977000	78	.9965900
29	.9976000	79	.9972500
30	.9975000	80	.9978000
31	.9974000	81	.9982300
32	.9973000	82	.9985800
33	.9972000	83	.9988600
34	.9971000	84	.9991800
35	.9970000	85	.9995200
36	.9969000	86	.9998100
37	.9968000	87	.9999530
38	.9967000	88	.9999630
39	.9966000	89	.9999710
40	.9965000	90	.9999770
41	.9964000	91	.9999810
42	.9963000	92	.9999840
43	.9962000	93	.9999860
44	.9961000	94	.9999870
45	.9960000	95	.9999880
46	.9959000	96	.9999900
47	.9958000	97	1.0000000
48	.9957000	98	1.0000000
49	.9956000	99	1.0000000
50	.9955000	100	1.0000000
		101	1.0000000

APERTURE LENS
SPACING TO DIAMETER RATIO = 1.0

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.0260000
2	1.0000000	52	.0790000
3	1.0000000	53	.1500000
4	.9998700	54	.2230000
5	.9998460	55	.2960000
6	.9998100	56	.3690000
7	.9997740	57	.4390000
8	.9997380	58	.5080000
9	.9996400	59	.5730000
10	.9995500	60	.6340000
11	.9994470	61	.6900000
12	.9993150	62	.7410000
13	.9991500	63	.7850000
14	.9989500	64	.8240000
15	.9987000	65	.8560000
16	.9984000	66	.8830000
17	.9980100	67	.9050000
18	.9975400	68	.9230000
19	.9969500	69	.9380000
20	.9962000	70	.9500000
21	.9953000	71	.9600000
22	.9942000	72	.9667500
23	.9928000	73	.9739000
24	.9911000	74	.9790000
25	.9890000	75	.9831000
26	.9863000	76	.9863000
27	.9831000	77	.9890000
28	.9790000	78	.9911000
29	.9739000	79	.9928000
30	.9667500	80	.9942000
31	.9600000	81	.9953000
32	.9500000	82	.9962000
33	.9380000	83	.9969500
34	.9230000	84	.9975400
35	.9050000	85	.9980100
36	.8830000	86	.9984000
37	.8560000	87	.9987000
38	.8240000	88	.9989500
39	.7850000	89	.9991500
40	.7410000	90	.9993150
41	.6900000	91	.9994700
42	.6340000	92	.9995500
43	.5730000	93	.9996400
44	.5080000	94	.9997080
45	.4390000	95	.9997740
46	.3690000	96	.9998100
47	.2960000	97	.9998460
48	.2230000	98	.9998700
49	.1500000	99	1.0000000
50	.0790000	100	1.0000000
		101	1.0000000

APERTURE LENS
SPACING TO DIAMETER RATIO = 2.0

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.01400000
2	.9993450	52	.04200000
3	.9992750	53	.07600000
4	.9991200	54	.11900000
5	.9989200	55	.15800000
6	.9986800	56	.19800000
7	.9983900	57	.23800000
8	.9980300	58	.27900000
9	.9975900	59	.31900000
10	.9970400	60	.36000000
11	.9964000	61	.40200000
12	.9955700	62	.44440000
13	.9946000	63	.48700000
14	.9934000	64	.53000000
15	.9919500	65	.57300000
16	.9901200	66	.61500000
17	.9880000	67	.65700000
18	.9855000	68	.69900000
19	.9820000	69	.73900000
20	.9780000	70	.77500000
21	.9730000	71	.80900000
22	.9670000	72	.84000000
23	.9599000	73	.86700000
24	.9507000	74	.89000000
25	.9400000	75	.91000000
26	.9262000	76	.92620000
27	.9100000	77	.94000000
28	.8900000	78	.95070000
29	.8670000	79	.95990000
30	.8400000	80	.96700000
31	.8090000	81	.97300000
32	.7750000	82	.97800000
33	.7380000	83	.98200000
34	.6990000	84	.98530000
35	.6570000	85	.98900000
36	.6150000	86	.99012000
37	.5730000	87	.99195000
38	.5300000	88	.99340000
39	.4870000	89	.99460000
40	.4440000	90	.99557000
41	.4020000	91	.99640000
42	.3600000	92	.99704000
43	.3190000	93	.99759000
44	.2790000	94	.99803000
45	.2380000	95	.99839000
46	.1980000	96	.99868000
47	.1580000	97	.99892000
48	.1190000	98	.99912000
49	.0760000	99	.99927500
50	.0420000	100	.99934500
		101	1.00000000

EINZEL LENS
 SPACING TO DIAMETER RATIO = 0.6
 MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.1

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.3620000
2	.9999450	52	.3664000
3	.9999310	53	.3794000
4	.9999120	54	.4004000
5	.9998900	55	.4282000
6	.9998600	56	.4617000
7	.9998250	57	.4995000
8	.9997800	58	.5405000
9	.9997200	59	.5836000
10	.9996500	60	.6276000
11	.9995550	61	.6716000
12	.9994400	62	.7144000
13	.9992900	63	.7552000
14	.9991000	64	.7928000
15	.9988600	65	.8269000
16	.9985600	66	.8569000
17	.9982000	67	.8829000
18	.9977000	68	.9050000
19	.9970700	69	.9234000
20	.9963000	70	.9385000
21	.9953000	71	.9509000
22	.9941000	72	.9610000
23	.9925000	73	.9691000
24	.9905000	74	.9758000
25	.9879000	75	.9807000
26	.9847000	76	.9847000
27	.9807000	77	.9879000
28	.9758000	78	.9905000
29	.9691000	79	.9925000
30	.9610000	80	.9941000
31	.9509000	81	.9953000
32	.9385000	82	.9963000
33	.9234000	83	.9970700
34	.9050000	84	.9977000
35	.8829000	85	.9982000
36	.8569000	86	.9985600
37	.8269000	87	.9988600
38	.7928000	88	.9991000
39	.7552000	89	.9992900
40	.7144000	90	.9994400
41	.6716000	91	.9995550
42	.6276000	92	.9996500
43	.5836000	93	.9997200
44	.5405000	94	.9997800
45	.4995000	95	.9998250
46	.4617000	96	.9998600
47	.4282000	97	.9998900
48	.4004000	98	.9999120
49	.3794000	99	.9999310
50	.3664000	100	.9999450
		101	1.0000000

EINZEL LENS
 SPACING TO DIAMETER RATIO = 0.6
 MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.2

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.2986000
2	1.0000000	52	.3160000
3	1.0000000	53	.3654000
4	1.0000000	54	.4383000
5	1.0000000	55	.5254000
6	1.0000000	56	.6181000
7	1.0000000	57	.7084000
8	1.0000000	58	.7891000
9	1.0000000	59	.8546000
10	1.0000000	60	.9035000
11	1.0000000	61	.9376000
12	1.0000000	62	.9605000
13	1.0000000	63	.9752000
14	1.0000000	64	.9845000
15	1.0000000	65	.9903000
16	1.0000000	66	.9939000
17	1.0000000	67	.9962000
18	1.0000000	68	.9976000
19	1.0000000	69	.9985000
20	1.0000000	70	.9990300
21	1.0000000	71	.9994100
22	.9999900	72	.9996700
23	.9999848	73	.9997690
24	.9999760	74	.9998520
25	.9999620	75	.9999060
26	.9999400	76	.9999400
27	.9999060	77	.9999620
28	.9998520	78	.9999760
29	.9997690	79	.9999848
30	.9996700	80	.9999900
31	.9994100	81	1.0000000
32	.9990300	82	1.0000000
33	.9985000	83	1.0000000
34	.9976000	84	1.0000000
35	.9962000	85	1.0000000
36	.9939000	86	1.0000000
37	.9903000	87	1.0000000
38	.9845000	88	1.0000000
39	.9752000	89	1.0000000
40	.9605000	90	1.0000000
41	.9376000	91	1.0000000
42	.9035000	92	1.0000000
43	.8546000	93	1.0000000
44	.7891000	94	1.0000000
45	.7084000	95	1.0000000
46	.6181000	96	1.0000000
47	.5254000	97	1.0000000
48	.4383000	98	1.0000000
49	.3654000	99	1.0000000
50	.3160000	100	1.0000000
		101	1.0000000

EINZEL LENS
 SPACING TO DIAMETER RATIO = 0.6
 MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.4

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.1978000
2	1.0000000	52	.2132000
3	1.0000000	53	.2576000
4	1.0000000	54	.3269000
5	1.0000000	55	.4137000
6	1.0000000	56	.5099000
7	1.0000000	57	.6084000
8	1.0000000	58	.7024000
9	1.0000000	59	.7853000
10	1.0000000	60	.8522000
11	1.0000000	61	.9021000
12	1.0000000	62	.9367000
13	1.0000000	63	.9599000
14	1.0000000	64	.9748000
15	1.0000000	65	.9843000
16	1.0000000	66	.9902000
17	1.0000000	67	.9939000
18	1.0000000	68	.9962000
19	1.0000000	69	.9976000
20	1.0000000	70	.9985000
21	.9999900	71	.9990500
22	.9999854	72	.9994000
23	.9999770	73	.9996300
24	.9999630	74	.9997660
25	.9999420	75	.9998520
26	.9999070	76	.9999070
27	.9998520	77	.9999420
28	.9997660	78	.9999630
29	.9996300	79	.9999770
30	.9994000	80	.9999854
31	.9990500	81	.9999900
32	.9985000	82	1.0000000
33	.9976000	83	1.0000000
34	.9962000	84	1.0000000
35	.9939000	85	1.0000000
36	.9902000	86	1.0000000
37	.9843000	87	1.0000000
38	.9748000	88	1.0000000
39	.9599000	89	1.0000000
40	.9367000	90	1.0000000
41	.9021000	91	1.0000000
42	.8522000	92	1.0000000
43	.7853000	93	1.0000000
44	.7024000	94	1.0000000
45	.6084000	95	1.0000000
46	.5099000	96	1.0000000
47	.4137000	97	1.0000000
48	.3269000	98	1.0000000
49	.2576000	99	1.0000000
50	.2132000	100	1.0000000
		101	1.0000000

EINZEL LENS
 SPACING TO DIAMETER RATIO = MUCH LESS THAN 1.0
 MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.1515

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.8026000
2	1.0000000	52	.8100000
3	1.0000000	53	.8302000
4	1.0000000	54	.8580000
5	1.0000000	55	.8877000
6	1.0000000	56	.9152000
7	1.0000000	57	.9381000
8	1.0000000	58	.9560000
9	1.0000000	59	.9693000
10	1.0000000	60	.9788000
11	1.0000000	61	.9855000
12	1.0000000	62	.9902000
13	1.0000000	63	.9934000
14	1.0000000	64	.9955000
15	1.0000000	65	.9970000
16	1.0000000	66	.9979700
17	1.0000000	67	.9986200
18	1.0000000	68	.9990600
19	1.0000000	69	.9993650
20	1.0000000	70	.9995700
21	1.0000000	71	.9997100
22	.9999910	72	.9998020
23	.9999868	73	.9998660
24	.9999806	74	.9999090
25	.9999717	75	.9999380
26	.9999580	76	.9999580
27	.9999380	77	.9999717
28	.9999090	78	.9999806
29	.9998660	79	.9999868
30	.9998020	80	.9999910
31	.9997100	81	1.0000000
32	.9995700	82	1.0000000
33	.9993650	83	1.0000000
34	.9990600	84	1.0000000
35	.9986200	85	1.0000000
36	.9979700	86	1.0000000
37	.9970000	87	1.0000000
38	.9955000	88	1.0000000
39	.9934000	89	1.0000000
40	.9902000	90	1.0000000
41	.9855000	91	1.0000000
42	.9788000	92	1.0000000
43	.9693000	93	1.0000000
44	.9560000	94	1.0000000
45	.9381000	95	1.0000000
46	.9152000	96	1.0000000
47	.8877000	97	1.0000000
48	.8580000	98	1.0000000
49	.8302000	99	1.0000000
50	.8100000	100	1.0000000
		101	1.0000000

EINZEL LENS
 SPACING TO DIAMETER RATIO = MUCH LESS THAN 1.0
 MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.303

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.6200000
2	1.0000000	52	.6328000
3	1.0000000	53	.6680000
4	1.0000000	54	.7179000
5	1.0000000	55	.7732000
6	1.0000000	56	.8258000
7	1.0000000	57	.8712000
8	1.0000000	58	.9074000
9	1.0000000	59	.9348000
10	1.0000000	60	.9548000
11	1.0000000	61	.9690000
12	1.0000000	62	.9789000
13	1.0000000	63	.9857000
14	1.0000000	64	.9904000
15	1.0000000	65	.9935000
16	1.0000000	66	.9955400
17	1.0000000	67	.9970500
18	1.0000000	68	.9980000
19	1.0000000	69	.9986600
20	1.0000000	70	.9991000
21	.9999875	71	.9993900
22	.9999818	72	.9995900
23	.9999732	73	.9997200
24	.9999605	74	.9998120
25	.9999420	75	.9998730
26	.9999140	76	.9999140
27	.9998730	77	.9999420
28	.9998120	78	.9999605
29	.9997200	79	.9999732
30	.9995900	80	.9999818
31	.9993900	81	.9999875
32	.9991000	82	1.0000000
33	.9986600	83	1.0000000
34	.9980000	84	1.0000000
35	.9970500	85	1.0000000
36	.9955400	86	1.0000000
37	.9935000	87	1.0000000
38	.9904000	88	1.0000000
39	.9857000	89	1.0000000
40	.9789000	90	1.0000000
41	.9690000	91	1.0000000
42	.9548000	92	1.0000000
43	.9348000	93	1.0000000
44	.9074000	94	1.0000000
45	.8712000	95	1.0000000
46	.8258000	96	1.0000000
47	.7732000	97	1.0000000
48	.7179000	98	1.0000000
49	.6680000	99	1.0000000
50	.6328000	100	1.0000000
		101	1.0000000

EINZEL LENS

SPACING TO DIAMETER RATIO = MUCH LESS THAN 1.0

MID-CYLINDER LENGTH TO DIAMETER RATIO = 0.606

Z	V(NORM)	Z	V(NORM)
1	1.0000000	51	.3360000
2	1.0000000	52	.3507000
3	1.0000000	53	.3932000
4	1.0000000	54	.4586000
5	1.0000000	55	.5392000
6	1.0000000	56	.6253000
7	1.0000000	57	.7080000
8	1.0000000	58	.7860000
9	1.0000000	59	.8402000
10	1.0000000	60	.8863000
11	1.0000000	61	.9205000
12	1.0000000	62	.9452000
13	1.0000000	63	.9625000
14	1.0000000	64	.9745000
15	1.0000000	65	.9827000
16	1.0000000	66	.9884000
17	1.0000000	67	.9921000
18	.9999900	68	.9947000
19	.9999860	69	.9964500
20	.9999790	70	.9976000
21	.9999690	71	.9984000
22	.9999540	72	.9989800
23	.9999320	73	.9992700
24	.9999000	74	.9995100
25	.9998500	75	.9996700
26	.9997790	76	.9997790
27	.9996700	77	.9998500
28	.9995100	78	.9999000
29	.9992700	79	.9999520
30	.9989800	80	.9999540
31	.9984000	81	.9999690
32	.9976000	82	.9999790
33	.9964500	83	.9999860
34	.9947000	84	.9999900
35	.9921000	85	1.0000000
36	.9884000	86	1.0000000
37	.9827000	87	1.0000000
38	.9745000	88	1.0000000
39	.9625000	89	1.0000000
40	.9452000	90	1.0000000
41	.9205000	91	1.0000000
42	.8863000	92	1.0000000
43	.8402000	93	1.0000000
44	.7860000	94	1.0000000
45	.7080000	95	1.0000000
46	.6253000	96	1.0000000
47	.5392000	97	1.0000000
48	.4586000	98	1.0000000
49	.3932000	99	1.0000000
50	.3360000	100	1.0000000
		101	1.0000000

APPENDIX II
CHROMATIC ABERRATION PROGRAM

Program CHROMAX is printed here in its entirety, liberally sprinkled with comment cards so that it can be easily modified for use on other computers such as the IBM 7094. It is followed by the input data requirements and a sample printout. If one desired a printout of the array of the ray trace and its derivatives, they would appear just above the words "completion of integration".


```

PROGRAM CHROMAX
ODIMENSION V(101),VI(1001),VPR(101),VPRI(1001),VDPR(101),VDPRI
1(1001),R(30),ROOT(30),C(15),RDATA(101),NTITLE(5),IDATE(5),
2RFIN(100),ROOTN(100),ITITLE(10),IDENT1(5),IDENT2(8),MCL(5)
COMMON NR,NRC,NSUBR,MRC,NRUN,ITITLE,RFIN,ROOTN,DIAM,KPRINT
NRYR = 0

```

- * READ THE ORIGINATORS IDENTIFICATION, THE DATE, AND THE NUMBER OF LENS CONFIGURATIONS TO BE STUDIED.


```

100 READ 100,(ITITLE(I),I=1,5),(IDATE(I),I=1,5),NXR
      FORMAT(5A8/5A8/I2)

```
- * READ THE IDENTIFICATION OF THE LENS.


```

5 READ 5,(IDENT1(I),I=1,9),(IDENT2(I),I=1,9),KPRINT,DIAM
      FORMAT(9A8/9A8/I2,F10.0)

```
- * READ THE NORMALIZED VOLTAGE DATA FOR THE LENS.


```

11 READ 11,(RDATA(I),I=1,101)
      FORMAT(5F15.8)

```
- * INITIALIZE CONTROLS BEFORE BEGINNING CALCULATIONS.


```

NRUN=1
8 C(10)=1.0
      NRYR=NRYR+1

```
- * PRINT THE TEST SERIES, THE DATE, AND THE LENS IDENTIFICATION.


```

PRINT 3333,NRYR,(IDATE(I),I=1,5),(IDENT1(I),I=1,9),(IDENT2(I),
      I=1,9)
3333 FORMAT(1H1,3X,21HCHRMX TEST SERIES NO ,I2,3X,5A8/5X,9A8/5X,9A8)

```
- * ON THE FIRST PASS WITH A NEW LENS GO TO BEVRAC.


```

      GO TO 10

```
- * AFTER CALLING BEVRAC FOR FIRST TIME WITH EACH LENS RETURN TO THIS POINT.


```

* BEGIN CALCULATIONS BY OBTAINING AXIAL VOLTAGE DATA ARRAY.

```
- * SUBROUTINE DATA CALCULATES REQUIRED V DATA.SEE SUBR DATA BELOW.


```

9 CALL DATA(V,VPR,VDPR,RDATA,C)
      MRC=1

```
- * SUBROUTINE INTERP PROVIDES 1001 DATA POINTS GIVEN 101 VALUES.


```

      CALL INTERP(V,VI)
      CALL INTERP(VPR,VPRI)
      CALL INTERP(VDPR,VDPRI)

```
- * SUBROUTINE BEVRAC CONSTITUTES THE HEART OF THE PROGRAM . IT


```

* TRACES THE RAYS THROUGH THE FIELD OF THE LENS TO OBTAIN THE
* TRAJECTORIES NECESSARY TO CALCULATE CIRCLE OF CONFUSION
* RESULTING FROM CHROMATIC ABERRATION. BEVRAC READS THE INITIAL CON-
* DITIONS, CYLINDER VOLTAGES, AND ADDITIONAL CONTROLS WHEN CALLED.

```
- 10 CALL BEVRAC(Z,R,ROOT,C)
- * COMPUTED GO TO STATEMENT ROUTES PATH UPON RETURN FROM BEVRAC.


```

      MRC=1
      2 NEW VOLTAGE DATA REQUIRED - CALL CONSTAB.
      3 INTEGRATION COMPLETE - CALL CONCIRC
      4 LENS CONFIGURATION COMPLETE - CHECK ON TEST RUNS COMPLETED
      AGAINST NXR. TERMINATE OR CONTINUE.

```
- GO TO (21,9,12,88),MRC

* SUBROUTINE CONSTAB CALCULATES THE CONSTANTS REQUIRED FOR INTEGRATION.

21 CALL CONSTAB(VI,VPRI,VDPRI,C,Z,IZ)

* THE PARAXIAL RAY EQUATION IS WRITTEN IN THE FORM REPRESENTED BY THE
* TWO EQUATIONS BELOW FOR NUMERICAL INTEGRATION USING THE
* RUNGE-KUTTA TECHNIQUE.

22 RDOT(1)=R(2)
RDOT(2)=-R(2)*C(6)-R(1)*C(7)

* C(11) CONTROLS THE FREQUENCY OF ARRAY DATA POINT PRINT OUT.

C(11)=50.0
GO TO 10
88 NXR=NXR-1

* IF NXR IS ZERO THE PROGRAM IS TERMINATED OTHERWISE DATA FOR A NEW
* LENS IS READ IN AND THE PROGRAM REPEATED.

IF(NXR)99,99,4

* SUBROUTINE CONCIRC PERFORMS CHROMATIC ABERRATION CALCULATIONS REQ.

12 CALL CONCIRC
IF(NRC-NR)43,33,33
33 C(10)=9.0
GO TO 10
43 C(10)=8.0
GO TO 10
99 END

SUBROUTINE INTERP(RDATA,RDI)

* SUBROUTINE INTERP INTERPOLATES LINEARLY BETWEEN THE POINTS OBTAINED
 * FOR THE AXIAL VOLTAGE AND ITS FIRST TWO DERIVATIVES IN
 * SUBROUTINE DATA TO OBTAIN AN ARRAY OF 1001 FOR EACH FROM THE
 * GENERATED 101 VALUE ARRAYS.

```

    DIMENSION RDATA(101),RDI(1001)
    DO 25 K=1,100
    DO25 I=1,10
    AI=1
    M=10*(K-1)+I
25  RDI(M)=RDATA(K)+0.1*(AI-1.)*(RDATA(K+1)-RDATA(K))
    RDI(1001)=RDATA(101)
    RETURN
  END
  
```

SUBROUTINE CONSTAB(XR,XPR,XDPR,C,Z,IZ)

* THIS SUBROUTINE CALCULATES THE CONSTANTS REQUIRED BY THE PARAXIAL RAY
 * EQUATION AT EACH POINT IN THE STEP BY STEP INTEGRATION PROCESS.
 * FOR EACH STRIDE THAT RKUTTA2 TAKES, CONSTAB MUST BE CALLED TO PROVIDE
 * THE NECESSARY CONSTANTS.

* THE CONSTANT C(1) IS THE INITIAL VELOCITY.
 * C(6,7) ARE THE CONSTANTS OF INTEGRATION IN BEVRAC.

```

    DIMENSION XR(1001),XPR(1001),XDPR(1001),C(15)
    IZ=Z
    VNX=XR(IZ)+C(1)
    C(6)=0.05*XPR(IZ)/VNX
    C(7)=0.0025*XDPR(IZ)/VNX
82  RETURN
  END
  
```


SUBROUTINE DATA(V,VPR,VDPR,RDATA,C)

- * COMPUTES AXIAL VOLTAGE AND ITS FIRST TWO DERIVATIVES FROM NORMALIZED
- * AXIAL VOLTAGE DATA FOR A GIVEN LENS CONFIGURATION.

DIMENSION RDATA(101),V(101),VPR(101),VDPR(101),C(15)

- * THE CONSTANTS C(2,3,4) DEFINE THE CYLINDER VOLTAGES.

V1=C(2)

V2=C(3)

- * TWO ROUTES THROUGH THE SUBROUTINE EXIST AT THIS POINT
- * DEPENDING ON WHETHER THE LENS CONFIGURATION HAS TWO OR THREE
- * CYLINDERS.

IF(C(4))78,69,78

- * FOR THE THREE CYLINDER LENS.
- * THE LENS IS DIVIDED AT ITS POINT OF SYMMETRY AND THE VOLTAGE DATA
- * IS CALCULATED FOR EACH HALF.

78 V3=C(4)

DELV = V1-V2

- * FOR THE INTERVAL Z=1,51 THE FOLLOWING PROCEDURE IS FOLLOWED.

DO 17 I =1,51

17 V(I)=DELV*RDATA(I)+V2

DELV=V3-V2

DO 18 I =52,101

18 V(I)=DELV*RDATA(I)+V2

DELV = V1-V2

VPR(1)=0.0

DO 27 J=2,51

27 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))

DELV = V3-V2

- * FOR THE INTERVAL Z=52,101 THE FOLLOWING.

DO 28 J=52,100

28 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))

GO TO 26

- * FOR THE TWO CYLINDER LENS.

- * THE VOLTAGE DATA IS CALCULATED FOR THE INTERVAL Z=1,101 DIRECTLY.

69 DELV = V2-V1

DO 15 I = 1,101

15 V(I)=DELV*RDATA(I)+V1

VPR(1) = 0.0

VPR(101) = 0.0

DO 25 J = 2,100

25 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))

26 VDPR(1) = 0.0

VDPR(101) = 0.0

DO 35 K = 2,100

35 VDPR(K) = 0.5*(VPR(K+1)-VPR(K-1))

RETURN

END

SUBROUTINE CONCIRC

```

* SUBROUTINE CONCIRC CALCULATES THE CHROMATIC ABERRATION COEFFICIENT,
* THE FOCAL LENGTH OF THE LENS WITH THE OBJECT BEFORE THE LENS,
* AND THE LOCATION OF THE CIRCLE OF LEAST CONFUSION.

  DIMENSION RFIN(100),RDOTN(100),ITITLE(10)
  COMMON NR,NRC,NSUBR,MRC,NRUN,ITITLE,RFIN,RDOTN,DIAM,KPRINT
  PRINT 675
675 FORMAT(///,57H CHROMATIC ABERRATION DUE TO SPREAD OF VELOCITIES O
1 F 1 EV //)
  I = NRC-NSUBR + 1
  K = 1+I
  ZNUM = RFIN(I) + RFIN(K)
  ZDEN = RDOTN(I) + RDOTN(K)
  Z = -0.1*ZNUM/ZDEN

* CALCULATE THE CHROMATIC ABERRATION COEFFICIENT.PRINT.

  D=-(10.*Z*RDOTN(I)+RFIN(I))*DIAM
  PRINT 13,D
13 FORMAT(41H CHRCMATIC ABERRATION COEFFICIENT = 1PE9.3)

* THIS COEFFICIENT WHEN MULTIPLIED BY THE INITIAL SPREAD OF VELOCITIES
* DIVIDED BY THE ACCELERATING VOLTAGE IN KV AND MULTIPLIED BY THE
* SIZE OF THE APERTURE WILL GIVE THE RADIIJS OF THE CIRCLE OF LEAST CONF.

* CALCULATE THE AXIAL LOCATION OF THE CIRCLE OF LEAST CONF. PRINT.

  Z=(Z+50.0)/DIAM
  PRINT 6543
6543 FORMAT(//,24H DISTANCES IN DIAMETERS //)
  PRINT 14,Z
14 FORMAT(41H LOCATION OF CIRCLE OF LEAST CONFUSION = F8.3,17H FROM L
1ENS CENTER )

* CALCULATE THE FOCAL LENGTH.PRINT.

  FD = -0.1/(RDOTN(I)*DIAM)
  PRINT 20,FD
20 FORMAT(16H FOCAL LENGTH = F8.3, //)
  END

```


SUBROUTINE PEVRAC (TC, XC, LX, C)

** SUBROUTINE USES FOURTH-ORDER RUNGE-KUTTA METHOD TO INTEGRATE
 ** ORDINARY DIFFERENTIAL EQUATIONS SUPPLIED BY THE CALLING
 ** PROGRAM. SINCE THESE EQUATIONS MUST BE REPEATEDLY QUOTE
 ** CALLED UNQUOTE BY THIS PROGRAM, A COMPUTED GO TO IS HERE
 ** USED TO ENSURE RETURN TO THE CORRECT PART OF THIS SUBROUTINE.

DIMENSION X(30), DX(30), C(15), XC(30), ITITLE(10), JTITLE(10),
 IIP(10), IG(10), PR(10), GR(10), RFIN(100), RDOTN(100)
 COMMON NR, NRC, NSUBR, MRC, NRUN, ITITLE, RFIN, RDOTN, DIAM, KPRINT
 1003 INDIC=C(10)

** THE FOLLOWING COMPUTED GO TO STATEMENT CONTROLS THE POINT OF ENTRY
 ** WHEN BEVRAC IS CALLED. STATEMENTS BELOW BRIEFLY DESCRIBE THE
 ** SECTIONS OF BEVRAC AS SEPARATED.

** GO TO (1,2000,50,62,88,88,423,1000,331),INDIC
 ** INDIC=1 READ ADDITIONAL LENS DATA AND CONTROLS. INITIALIZE.
 ** 2 ENTER MAIN LOOP - BEGIN INTEGRATION. PRINT ARRAY COLUMN
 ** HEADINGS IF KPRINT.
 ** 3 DEFINE ARRAY VARIABLES FOR PRINTING. PRINT IF KPRINT.
 ** 4 CHECK STATUS OF INTEGRATION - CONTINUE OR TERMINATE.
 ** 5 CALL RKUTTA2 - NEXT STRIDE.
 ** 6 CALL RKUTTA2 - SECOND HALF OF STRIDE.
 ** 7 PRINT INITIAL CONDITIONS AND INPUT PARAMETERS. DEFINE
 ** JJ TO BE THE NUMBER COLUMNS FOR ARRAY PRINTING.
 ** 8 CHECK NO OF RUNS COMPLETED AGAINST NO OF SUBRUNS WITH
 ** DIFFERENT INITIAL CONDITIONS. ADJUST CONTROLS. PRINT
 ** PAGE INITIAL DATA.
 ** 9 TERMINATE THE RUN - PRINT.
 **
 ** READ DATA AND PRINT RECCRD.

** NR DEFINES THE NUMBER OF VOLTAGE RUNS. NSUBR DEFINES THE NUMBER
 ** OF SUB RUNS WITH INITIAL VELOCITY CHANGE.

1 READ 101,NR,NSUBR
 101 FORMAT(2I2)
 NN=2
 NRC = 0

** READ THE INITIAL VALUE OF Z, THE STEP SIZE, AND THE FINAL VALUE OF Z.

7 READ 103, TI, DT, TF
 1000 NRC = NRC + 1
 IF(NRC-1)1111,1100,1111
 1111 IF(XMOD(NRC-1,NSUBR))1100,1122,1100
 1122 PRINT 1133
 1133 FORMAT(1H1)
 1100 PRINT 201, (ITITLE(I), I=1,5)
 IF (NRC - 1)6,3,6
 3 IF (NR - 1)4,5,4
 4 NNN=NR/NSUBR
 PRINT 202,NNN
 202 FORMAT (/,37X,12,20H RUNS ARE CALLED FOR)
 GO TO 6
 5 PRINT 203
 GO TO 12
 6 IF(XMOD(NRC-1,NSUBR))12,2,12
 2 PRINT 204,NRUN
 201 FORMAT (////,36X,5A8)
 203 FORMAT (/,37X,21H ONE RUN IS CALLED FOR ,/,18H INPUT DATA RECORD)
 204 FORMAT (///,34H INPUT DATA RECORD FOR RUN NUMBER ,12)

** THE CONSTANT C(1) IS THE INITIAL VELOCITY IN VOLTS.
 ** C(2,3,4) ARE THE ELECTRODE POTENTIALS.
 ** C(5) IS A VOLTAGE DATA CONTROL VARIABLE.
 ** C(6,7) ARE CONSTANTS DEFINED BY SUBR CONSTAB.
 ** C(8) IS AN INITIAL CONDITION CHANGE PARAMETER.
 ** C(9) NOT USED.
 ** C(10) IS INDIC A CONTROL PARAMETER.
 ** C(11) CONTROLS ARRAY PRINTOUT STEP SIZE.

12 READ 103, (C(I), I=1,8)
 103 FORMAT(7F10.4,F7.4)


```

4008 IF(C(8))4008,4007,4008
      READ 103,(X(1),1=1,NN)
      RINIT = X(1)
      ROOTIN = X(2)
4007 J=0
      X(1) = RINIT
      X(2) = ROOTIN
      DO 14 I=1,8
        IF(C(1))13,14,13
      13 J = J + 1
      14 CONTINUE
      K = 0
      DO 16 I=1,NN
        IF(X(1))15,16,15
      15 K = K + 1
      16 CONTINUE
      19 PRINT 211
2001 PRINT 2005,C(1)
420 DO 422 I=2,4
      IF(C(I))421,422,421
      421 GO TO (2001,2002,2003,2004),I
2005 FORMAT(3X,19HINITIAL VELCCITY = ,E10.4)
      GO TO 422
2002 PRINT 2006,C(I)
2006 FORMAT(17X,5HV1 = ,F10.1)
      GO TO 422
2003 PRINT 2007,C(I)
2007 FORMAT(17X,5HV2 = ,F10.1)
      GO TO 422
2004 PRINT 2008,C(I)
2008 FORMAT(17X,5HV3 = ,F10.1)
      GO TO 422
      422 CONTINUE
      IF(C(5))423,423,1001
1001 MRC=2
      NRUN=NRUN+1
      C(10)=7.0
      RETURN
      211 FORMAT(/,25H THE INPUT PARAMETERS ARE )
      212 FORMAT(14X,2HC(,I2,4H) = ,E10.4)

423 PRINT 1211
427 DO 429 I=1,NN
428 GO TO (3001,3002),I
3001 PRINT 3004,X(I)
3004 FORMAT(18X4HR = ,F10.1)
      GO TO 429
3002 PRINT 3005,X(I)
3005 FORMAT(15X7HRDOT = ,F10.1)
      GO TO 429
      429 CONTINUE
1211 FORMAT(/,27H THE INITIAL CONDITCNIS ARE )
      20 IF(C(8))4020,4021,4020
4020 READ 104,(JTITLE(I), IP(I), I = 1,8)
      104 FORMAT(8(A8,I2))
4021 IBLANK=8H
      DO 21 J=1,8
        IF(JTITLE(J) - IBLANK)21,22,21
      21 CONTINUE
      J = 9
      22 JJ = J - 1

306 CONTINUE

```

* THIS ENDS THE BOOK-KEEPING. INITIALIZE BEFORE ENTERING MAIN LOOP.


```

31 IPAGE = 0
   T = T1
   NOPTS = 0
41 C(11) = 20.
   C(13) = DT
   DO 42 I=1,NN
42 XC(I) = X(I)
   TC = T
   C(10) = 2.
   RETURN

2000 IF(JJ)43,62,43
43 INCPR = C(11)
   C(11) = 20.
   IF(XMCOF(NOPTS,50*INCPR))45,1046,45
45 IF(XMCOF(NOPTS, INCPR))56,48,56
1049 PRINT 218,(JTITLE(I),I=1,8)
1046 IF(KPRINT)1049, 47,1049
1047 PRINT 219
   PRINT 219
   GO TO 47
   PRINT 219
47 PRINT 219
218 FORMAT(///,11X,8(A8,5X))
219 FORMAT(1H )
48 DO 49 I=1,NN
49 XC(I) = X(I)
   TC = T
   C(10) = 3.
   RETURN

50 DO 53 I=1,JJ
   IF(IP(I))52,51,52
51 PR(I) = T
   GO TO 53
52 IPI = IP(I)
   PR(I) = XC(IPI)
53 CONTINUE
   IF(KPRINT)1048,56,1048
1048 PRINT 220, (PR(I), I=1,JJ)
220 FORMAT (7X, 8E13.5)
56 DO 57 I=1,NN
57 XC(I) = X(I)
   TC = T
   C(10) = 4.
   RETURN

62 NOPTS = NOPTS + 1
64 IF(NOPTS - 4500)66,65,66
65 PRINT 222
222 FORMAT (////////,31H STOP AT 4500 INTEGRATION STEPS )
   GO TO 350
66 DO 70 I=1,NN
   IF(ABSF(X(I)) - 1.0E+12)70,70,71
70 CONTINUE
   GO TO 72
71 PRINT 224
224 FORMAT (////////,76H STOP WITH THE ABSOLUTE VALUE OF A DEPENDENT VAR
1TABLE GREATER THAN 1.0E+12. ,/,57H INTEGRATION PROBABLY UNSTABLE.
2 TRY A SMALLER STEP SIZE. ,26HNO GRAPHS WILL BE PLOTTED. )
   GO TO 330
72 DT = C(13)
73 IF(T - TF)87,74,74
74 PRINT 225
225 FORMAT ( ///,26H COMPLETION OF INTEGRATION )

```



```

271 RFIN(NRC)=PR(2)
    RDOTN(NRC)=PR(3)
    GO TO 330
87  C(10) = 5.
88  CALL RKUTTA2(NN, T, X, DT, C, TC, XC, DX)
    IF(C(10) - 6.)90,89,90
89  RETURN
90  T = T + DT
    GO TO 2000
330 IF(XMODF(NRC,NSUBR))335,334,335
334 MRC=3
    RETURN
335 IF(NRC - NR)1000,331,331
331 IF(NR - 1)333,332,333
332 PRINT 226
    MRC=4
    RETURN
333 PRINT 227,NNN
    MRC=4
    RETURN
226 FORMAT(///,43H THE ONE RUN CALLED FOR HAS BEEN COMPLETED.,//)
227 FORMAT(///,5H THE ,12,37H RUNS CALLED FOR HAVE BEEN COMPLETED.,//)
END

```


SUBROUTINE RKUTTA2(NN, T, X, DT, C, TC, XC, DX)

* RKUTTA2 INTEGRATES USING THE 4TH ORDER RUNGE-KUTTA TECHNIQUE.

DIMENSION X(30), C(15), XC(30), DX(30), CT(4), AK(4,30)
INDIC = C(10) - 4.0

* EACH STRIDE THAT RKUTTA2 TAKES IS COMPOSED OF TWO UNIT STEPSIZE
* INCREMENTS. INDIC BELOW ROUTES WHEN SUBROUTINE IS CALLED TO
* EITHER THE FIRST OR SECOND HALF STRIDE.

```
GO TO (1, 3), INDIC
1 CT(1) = 0.0
  CT(2) = 0.5
  CT(3) = 0.5
  CT(4) = 1.0
  DO 4 II=1, 4
    TC = T + CT(II)*DT
    DO 2 J=1, NN
      2 XC(J) = X(J) + CT(II)*AK(II-1, J)
      C(10) = 6.0
    RETURN
  3 DO 4 J=1, NN
    4 AK(II, J) = DT*DX(J)
    DO 5 J=1, NN
      5 X(J) = X(J) + (AK(1, J) + 2.*AK(2, J) + 2.*AK(3, J) + AK(4, J))/6.0
      C(10) = 7.0
    RETURN
  END
END
```


DATA REQUIRED FOR CHROMAX RUN WHICH FOLLOWS

BEGINNING WITH FIRST DATA CARD

CARDEN/TINKER/USNPGS...CHROMAX
5 APRIL 1963

1
IMMERSION LENS DIAM = 16.0 SPACING/DIAM RATIO = 2.0
SPACING = 100.0

16.0				
0.0	0.0	0.0	0.0000137	0.0000183
0.0000245	0.000033	0.000044	0.000059	0.0000810
0.000107	0.000144	0.000193	0.000255	0.00034
0.00046	0.00062	0.00084	0.0012	0.0015
0.0020	0.0027	0.0036	0.0048	0.0064
0.0086	0.0115	0.0153	0.0204	0.0270
0.0355	0.0465	0.0601	0.0768	0.0965
0.1188	0.1434	0.1696	0.1965	0.2237
0.2509	0.2777	0.3041	0.3299	0.3553
0.3801	0.4046	0.4287	0.4526	0.4763
0.5	0.5237	0.5474	0.5713	0.5954
0.6199	0.6447	0.6701	0.6959	0.7223
0.7491	0.7763	0.8035	0.8304	0.8566
0.8812	0.9035	0.9232	0.9399	0.9535
0.9645	0.9730	0.9796	0.9847	0.9885
0.9914	0.9936	0.9945	0.9964	0.9973
0.9980	0.9985	0.9988	0.99916	0.99938
0.99954	0.99966	0.999745	0.999807	0.999856
0.999893	0.99992	0.999941	0.999956	0.999967
0.9999755	0.9999817	0.9999863	1.0	1.0

1.0				
2.0				
1.0	2.0	1001.0		
0.0	1000.0	150.0	1.0	1.0
1.0				
7	00 R	01 RDOT	02	
1.0	1000.0	150.0		

CHRMX TEST SERIES NO 1 5 APRIL 1963
IMMERSION LENS DIAM = 16.0 SPACING/DIAM RATIO = 2.0
SPACING = 100.0

CARDEN/TINKER/USNPGS...CHROMAX

1 RUNS ARE CALLED FOR

INPUT DATA RECORD FOR RUN NUMBER 1

THE INPUT PARAMETERS ARE

INITIAL VELOCITY = .0000E+00
V1 = 1000.0
V2 = 150.0

THE INITIAL CONDITIONS ARE

R = 1.0
ROOT = .0

COMPLETION OF INTEGRATION

CARDEN/TINKER/USNPGS...CHROMAX

THE INPUT PARAMETERS ARE

INITIAL VELOCITY = .1000E+01
V1 = 1000.0
V2 = 150.0

THE INITIAL CONDITIONS ARE

R = 1.0
ROOT = .0

COMPLETION OF INTEGRATION

CHROMATIC ABERRATION DUE TO SPREAD OF VELOCITIES OF 1 EV

CHROMATIC ABERRATION COEFFICIENT = 7.250E-02

DISTANCES IN DIAMETERS

LOCATION OF CIRCLE OF LEAST CONFUSION = 4.565 FROM LENS CENTER
FOCAL LENGTH = 2.490

THE 1 RUNS CALLED FOR HAVE BEEN COMPLETED.

TIME, 1 MINUTES AND 9 SECONDS

APPENDIX III
SPHERICAL ABERRATION PROGRAM

Program SPHERAX is set up just like Appendix II for easy readability. It is geared to provide ray tracing print-out if desired. It will also provide data on lens cardinal points. Required data input and sample output follow the program. Note that all outputs that have the dimensions of length are printed out in "diameters". The "sphr, aber, coef." printed out is $Sg r_g^3$. The "nmalized sphr. aber. coef." printed out is the C_s that appears on the curves in Chapter V.


```

PROGRAM SPHERAX
DIMENSION Z(102),V(102),R(102),R1(102),DV(102),DR(102),DR1(102),
1D2V(102),D2R(102),D2R1(102),A(102),C(8),ITITLE(9),RDATA(101),PO(2)
2,P(5)
COMMON Z,R,DR,D2R

```

* READ THE NUMBER OF LENS CONFIGURATIONS TO BE STUDIED.

```

READ 1, NRUN
1 FORMAT(I2)
KN=1

```

* READ AND STORE DATA FOR LENS.

* READ TITLE AND CONTROL DATA FOR LENS CONFIGURATION.
 * READ FIRST CYLINDER DIAMETER AND APERTURE LENS CONTROL (IF REQUIRED).

```

922 READ 100, (ITITLE(I), I=1, 9), NVRUN, NDRUN, KPRINT, DIAM, APERT
100 FORMAT(9A8/3I2/2F10.8)
NVTEMP=NVRUN
NDTEMP=NDRUN

```

* THE CONSTANT C(1,2,3) ARE THE CYLINDER VOLTAGES.
 * C(4) IS THE VOLTAGE RATIO INCREASE FACTOR IN VOLTS.
 * C(5) IS THE OBJECT-IMAGE DISTANCE IN 1ST CYLINDER DIAMS.
 * C(6) IS OBJECT-IMAGE DISTANCE INCREASE FACTOR IN DIAMS.

```

555 READ 231, (C(I), I=1, 8)
231 FORMAT(8F10.4)

```

```

CTEMP=C(5)
DO 924 I=1,3
924 P(I)=C(I)
806 KV=1
KD=1
DISOI = C(5)*DIAM

```

* RDATA IS THE NORMALIZED VOLTAGE DATA FOR THE LENS.

```

READ 132, (RDATA(I), I=1, 101)
132 FORMAT(5F15.8)
926 PRINT 101
101 FORMAT(1H1)
DO 96 I=1,3
96 C(I)=P(I)
PRINT 136, KN
136 FORMAT(///, 5HLENS I2)
PRINT 100, (ITITLE(K), K=1, 9)
925 PRINT 135, KD
135 FORMAT(/, 3X, 5H RUN I2, 39H WITH CONSTANT OBJECT TO IMAGE DISTANCE
1, //)

```

* BEGIN CALCULATIONS BY OBTAINING AXIAL VOLTAGE DATA ARRAY.

* SUBROUTINE DATA CALCULATES REQUIRED V DATA.

```

923 CALL DATA(V, DV, D2V, RDATA, C)
PRINT 134, KV
134 FORMAT(///, 6X, 14H VOLTAGE RUN I2)

```

* PARAY TRACES THE PRINCIPAL RAYS TO OBTAIN OPTICAL PROPERTIES. PRINT.

* SET RADIUS AND INITIAL SLOPE FOR FIRST PRINCIPAL RAY.

```

124 R(1)=1.0
DR(1)=0.0
CALL PARAY(V, DV, D2V)
IF(DR(101)) 13, 4, 4

```

* IF F2 IS LESS THAN ZERO THE VOLTAGE RUN IS TERMINATED.


```

4 PRINT 106
106 FORMAT(1H1,49HF2 IS LESS THAN ZERO ...LENS TOO WEAK FOR IMAGING ,
1//)
3 GO TO 89
DO 2 I=1,101
R1(I)=R(I)
DR1(I)=DR(I)
2 D2R1(I)=D2R(I)
F2=-1.0/DR1(101)
P2=100.0-(R1(101)-1.0)/DR1(101)
ZLD=F2/DIAM
XLD=P2/DIAM
PRINT 107,ZLD,XLD
107 FORMAT(//,38H 2ND FOCAL LENGTH AND PRINC PLANE LCTN ,/,4H F2=F8.3
1 ,/,4H P2= F8.3 )

* SET RADIUS AND INITIAL SLOPE FOR SECCND PRINCIPAL RAY.

66 DR(1)=1.0
R(1)=0.0
CALL PARAY(V,DV,D2V)
6 F1=(R(101)*DR1(101)-R1(101)*DR(101))/DR1(101)
P1=(DR1(101)*R(101)-DR(101)*(R1(101)-1.0))/DR1(101)
ZLD=F1/DIAM
XLD=P1/DIAM
PRINT 109,ZLD,XLD
109 FORMAT(//,38H 1ST FOCAL LENGTH AND PRINC PLANE LCTN ,/,4H F1=F8.3
1 ,/,4H P1= F8.3 )

* SOLUTION OF PRINCIPAL RAYS IN ARRAY FORM ONLY PRINTED IF KPRINT IS +.

IF(KPRINT)51,38,51
51 PRINT 120
120 FORMAT(1H1,37HSOLUTION TC PARAY FOR PRINCIPAL RAYS )
PRINT 118
118 FORMAT(1H0,3X,1HZ,9X,1HV,11X,2HDV,11X,3HD2V,12X,2HR1,11X,3HDR1,13X
1,1HR,15X,2HDR)
PRINT 119,(Z(I),V(I),DV(I),D2V(I),R1(I),DR1(I),R(I),DR(I),I=1,101)
119 FORMAT(1H0,F6.2,2X,F9.2,8X,F6.2,5X,F10.5,1X,F10.5,5X,F10.5,5X,F10
1.5,5X,F10.5)

* BEGIN SPHERICAL ABERRATION CALCULATIONS.

* IF THE OBJECT-IMAGE DISTANCE IS NOT READ IN ITS CRITICAL VALUE IS
* CALCULATED. DISOI IN DIAMETERS IS PRINTED.

38 IF(C(5))927,927,39
927 DISOI=2.0*SQRT(F1*F2)+F2+F1+P2-P1
C(5)=DISOI
CTEMP=C(5)
39 PRINT 111
111 FORMAT(//,47H SOLUTION FOR SPHERICAL ABERRATION COEFFICIENTS //)
ZLD=DISOI/DIAM
PRINT 110,ZLD
110 FORMAT( ,28H OBJECT TO IMAGE DISTANCE = F8.3)
DISC=(DISOI-F2-P2+P1-F1)**2-4.0*F1*F2
IF(DISC+0.0001)10,11,11

* IF THE DISOI READ IN IS LESS THAN CRITICAL THE VOLTAGE RUN IS
* TERMINATED.

10 PRINT 121
121 FORMAT(35H DISOI SMALLER THAN CRITICAL VALUE ,//)
GO TO 89
11 B=DISOI-F2-P2-P1-F1
IF(DISC)865,36,36
865 PRINT 866
866 FORMAT(//,23H DISC LESS THAN ZERO ,/)
GO TO 89
36 SQDSC=SQRT(DISC)

```



```

* THE TWO POSSIBLE LENS POSITIONS DESCRIBED BY THE DISTANCE FROM OBJECT
* TO Z=1, PO(1) AND PO(2), ARE CALCULATED.
37 PO(1)=(B-SQRDSC)*0.5
   PO(2)=(B+SQRDSC)*0.5

* ENTER MAIN LOOP WHICH CALCULATES FOR EACH PO THE ABERRATION COEF.

   DO 28 J=1,2
   ZLD=PO(J)/DIAM
   PRINT 115,ZLD
115 FORMAT(1X,28H POSITION OF OBJECT           =   F8.3)
   PSP=(PO(J)+50.0)/DISOI
   PRINT 551,PSP
551 FORMAT(28H          POSITION PARAMETER =   F8.3)
   DO 60 I=1,101

* R ALPHA RAY WHICH IS A LINEAR COMBINATION OF TWO PRINCIPAL RAYS
* AND THE LENS POSITION, PO(J), IS CALCULATED IN ARRAY FORM.
   R(I)=PO(J)*R1(I)+R(I)
   D2R(I)= PO(J)*D2R1(I)+C2R(I)
60 DR(I)= PO(J)*DR1(I)+DR(I)
   IF(DR(101))939,936,936

* IF THE R ALPHA RAY DOES NOT CONVERGE STOP FOR PO(1) AND TRY PO(2).
936 PRINT 938
938 FORMAT(30H R ALPHA RAY DID NOT CONVERGE ,/)
   GO TO 28

* CALCULATE DEPENDING ON THE IMAGE POSITION,THE INTEGRAL REQUIRED FOR
* SPHERICAL ABERRATION COEF WITHIN THE FIELD AND ADD LINEAR EXTRAPOLA-
* TIONS FOR BOTH FIELD FREE REGIONS AS APPLICABLE.
939 IF(PO(J))30,12,12
12 IF(DISOI-PO(J)-100.0)13,13,14
13 Z1=1.0+DISOI-PO(J)
   GO TO 15
14 Z1=101.0
   T=0.5*SQRTF(V(101))*DR(101)**3*R(101)
15 NZ=Z1
   DO 18 I=1,60
   IF(NZ-I*2)16,17,18
16 NZ2=NZ+1
   GO TO 19
17 NZ2=NZ
   GO TO 19
18 CONTINUE
19 DO 20 I=1,NZ2
   R2=R(I)*R(I)
   R3=R2*R(I)
   R4=R3*R(I)
   DR2=DR(I)*DR(I)
   DR4=DR2*DR2
   DIV=D2V(I)/V(I)
   DIV2=DIV*DIV
   V2=V(I)*V(I)
   V3=V2*V(I)
20 A(I)=(SQRTF(V(I))/32.0)*(DIV*(4.*R2*DR2+4.*R3*D2R(I))-DIV2*(1.5*R4
1) -(DV(I)*D2V(I))/V2*(4.*R3*DR(I))+(DV(I)*DV(I)*D2V(I)/V3)*.75*R4-
216.*DR4)
   S=.5*(A(I)-A(NZ2))
   NZM1=NZ2-1
   DO 21 I=2,NZM1,2
21 S=S+2.*A(I)+A(I+1)
   S=.666667*S
   W=-0.5*SQRTF(V(I))*PO(J)
   IF(DISOI-PO(J)-100.0)22,22,23
22 S=S+W
   GO TO 24
23 S=S+W+T
24 IF(DISOI-PO(J)-100.0)25,25,26

```



```

* CALCULATE THE MAGNIFICATION, BMAG.PRINT.
25 BMAG=R1(NZ)
   GO TO 27
26 BMAG=R1(101)-DR1(101)*R(101)/DR(101)
27 PRINT 113,BMAG
113 FORMAT('28H MAGNIFICATION OF OBJECT = 1PE12.5)

* SOLVE FOR THE SPHERICAL ABERRATION CCEFICIENT.PRINT.
   G=ABSF(S*BMAG/SQRTF(V(1)))

* NORMALIZE ABOVE CALCULATED COEF USING THE FIRST CYLINDER DIAMETER AND
* LENS POSITION.PRINT.
   PN=R(101)/DIAM

* IF THE LENS IS APERTURE TYPE USE SPECIAL NORMALIZATION.
   IF(APERT)732,733,732
732 PN=R(51)/DIAM
733 GD8=G/(DIAM*PN**3)
   PRINT 114,G
114 FORMAT(8X,20H SPHER ABER COEF = 1PE12.5)
   PRINT 931,GD8
931 FORMAT('28H NMALIZED SPHER ABER COEF = 1PE12.5)

* SOLVE FOR THE VOLTAGE RATIO PARAMETER . PRINT.
   VD=C(1)-C(2)
   VS=C(1)+C(2)
   VRP=(VD/VS)*(VD/VS)
   PRINT 116,VRP
116 FORMAT(27H VOLTAGE RATIO PARAMETER = 1F8.3,/)
30 CONTINUE
31 DO 50 I=1,101
   R(I)=R(I)-PO(J)*R1(I)
   DR(I)=DR(I)-PO(J)*DR1(I)
50 D2R(I)=D2R(I)-PO(J)*D2R1(I)
28 CONTINUE

* REINITIALIZE ,SET CONTROLS,AND RETURN OR TERMINATE PROGRAM.

89 KV=KV+1
   NVRUN=NVRUN-1
90 C(2)=C(2)+C(4)
   IF(NVRUN)93,93,923
93 NDRUN=NDRUN-1
   NVRUN=NVTEMP
   KV=1
   IF(NDRUN)95,95,94
94 DISCI=DISOI+C(6)*DIAM
   KD=KD+1
   GO TO 926
95 NRUN=NRUN-1
   C(5)=CTEMP
   KD=1
   KN=KN+1
   NDRUN=NTEMP
921 PRINT 112
112 FORMAT('///,34H LENS CONFIGURATION STUDY COMPLETE ,/)
   IF(NRUN)99,99,922
99 CONTINUE
98 END

```


SUBROUTINE DATA(V,VPR,VDPR,RDATA,C)

* COMPUTES AXIAL VOLTAGE AND ITS FIRST TWO DERIVATIVES FROM NORMALIZED
* AXIAL VOLTAGE DATA FOR A GIVEN LENS CONFIGURATION.

DIMENSION RDATA(102),V(102),VPR(102),VDPR(102),C(8)
COMMON Z,R,DR,D2R

V1=C(1)
V2=C(2)

* TWO ROUTES THROUGH THE SUBROUTINE EXIST AT THIS POINT
* DEPENDING ON WHETHER THE LENS CONFIGURATION HAS TWO OR THREE
* CYLINDERS.

IF(C(3))78,69,78

* FOR THE THREE CYLINDER LENS.
* THE LENS IS DIVIDED AT ITS POINT OF SYMMETRY AND THE VOLTAGE DATA
* IS CALCULATED FOR EACH HALF.

78 V3=C(3)
DELV = V1-V2

* FOR THE INTERVAL Z=1,51 THE FOLLOWING PROCEDURE IS FOLLOWED.

DO 17 I =1,51
17 V(I)=DELV*RDATA(I)+V2
DELV=V3-V2

* FOR THE INTERVAL Z=52,101 THE FOLLOWING.

DO 18 I =52,101
18 V(I)=DELV*RDATA(I)+V2
DELV = V1-V2
VPR(1)=0.0
DO 27 J=2,51
27 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))
DELV =V3-V2
DO 28 J=52,100
28 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))
GO TO 26

* FOR THE TWO CYLINDER LENS.
* THE VOLTAGE DATA IS CALCULATED FOR THE INTERVAL Z=1,101 DIRECTLY.

69 DELV = V2-V1
DO 15 I = 1,101
15 V(I)=DELV*RDATA(I)+V1
VPR(1) = 0.0
VPR(101) = 0.0
DO 25 J = 2,100
25 VPR(J) = 0.5*DELV*(RDATA(J+1)-RDATA(J-1))
26 VDPR(1) = 0.0
VDPR(101) = 0.0
DO 35 K = 2,100
35 VDPR(K) = 0.5*(VPR(K+1)-VPR(K-1))
RETURN
END

SUBROUTINE PARAY(V,DV,D2V)

* TRACES RAYS THROUGH THE FIELD OF THE LENS. CALCULATES THE RADIAL
* POSITION AND ITS FIRST TWO DERIVATIVES.

```

1  DIMENSION Z(102),V(102),R(102),R1(102),DV(102),DR(102),DR1(102),
    D2V(102),D2R(102),D2R1(102),A(102),C(8),ITITLE(5),RDATA(101),PO(2)
    COMMON Z,R,DR,D2R
    Z(1)=1.
    V(102)=V(101)
    D2R(1)=-.5*(DR(1)*DV(1)+R(1)*D2V(1)*.5)/V(1)
    DO 3 I=2,101
    Z(I)=I
    R(I)=R(I-1)+DR(I-1)+.5*D2R(I-1)
    IF(2.*V(I)+DV(I))1,2,1
2  DR(I)=DR(I-1)+D2R(I-1)
    GO TO 6
1  DR(I)=.5*(DR(I-1)+D2R(I-1)+(DR(I-1)-R(I)*D2V(I)/(4.*V(I)))*2.*V(I)
    1/(2.*V(I)+DV(I)))
6  D2R(I)=- (2.*DV(I)*DR(I)+R(I)*D2V(I))/(4.*V(I))
3  CONTINUE
5  RETURN
    END
    END

```


DATA REQUIRED FOR SPHERAX RUN WHICH FOLLOWS

BEGINNING WITH FIRST DATA CARD

1	COAXIAL CYLINDER LENS (FLARED)	DIAMETER = 40.0		
2	1			
40.0				
100.0	400.0	50.0	40.0	
0.000	0.001	0.00115	0.0013	0.00145
0.0016	0.0018	0.00205	0.0023	0.0026
0.0029	0.0033	0.0037	0.0041	0.0046
0.0052	0.0058	0.0065	0.0073	0.0082
0.0092	0.0103	0.0115	0.0129	0.0145
0.0163	0.0183	0.0205	0.0230	0.0259
0.0290	0.0325	0.0364	0.0410	0.0460
0.0515	0.058	0.065	0.073	0.082
0.091	0.101	0.112	0.125	0.137
0.151	0.166	0.183	0.199	0.218
0.236	0.253	0.275	0.295	0.317
0.339	0.361	0.382	0.405	0.428
0.449	0.472	0.493	0.515	0.534
0.555	0.574	0.593	0.611	0.628
0.646	0.662	0.679	0.692	0.707
0.721	0.734	0.746	0.757	0.768
0.778	0.788	0.797	0.8055	0.814
0.823	0.830	0.838	0.845	0.852
0.858	0.864	0.870	0.876	0.882
0.887	0.892	0.896	0.901	0.905
0.909				

LENS 1
COAXIAL CYLINDER LENS (FLARED) DIAMETER = 40.0

RUN 1 WITH CONSTANT OBJECT TO IMAGE DISTANCE

VOLTAGE RUN 1

2ND FOCAL LENGTH AND PRINC PLANE LCTN

F2= 6.328

P2= -.326

1ST FOCAL LENGTH AND PRINC PLANE LCTN

F1= 3.283

P1= .191

SOLUTION FOR SPHERICAL ABERRATION COEFFICIENTS

OBJECT TO IMAGE DISTANCE = 40.000

POSITION OF OBJECT = .497

POSITION PARAMETER = .044

R ALPHA RAY DID NOT CONVERGE

POSITION OF OBJECT = 30.028

POSITION PARAMETER = .782

MAGNIFICATION OF OBJECT = -1.19672E-01

SPHER ABER COEF = 7.02114E+03

NORMALIZED SPHER ABER COEF = 2.78863E-02

VOLTAGE RATIO PARAMETER = .360

VOLTAGE RUN 2

2ND FOCAL LENGTH AND PRINC PLANE LCTN

F2= 5.587

P2= -.259

1ST FOCAL LENGTH AND PRINC PLANE LCTN

F1= 2.736

P1= .269

SOLUTION FOR SPHERICAL ABERRATION COEFFICIENTS

OBJECT TO IMAGE DISTANCE = 40.000
POSITION OF OBJECT = .213
POSITION PARAMETER = .037
R ALPHA RAY DID NOT CONVERGE

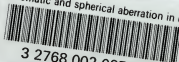
POSITION OF OBJECT = 31.454
POSITION PARAMETER = .818
MAGNIFICATION OF OBJECT = -9.37141E-02
SPHER ABER COEF = 9.64487E+03
NORMALIZED SPHER ABER COEF = 4.48692E-02
VOLTAGE RATIO PARAMETER = .405

LENS CONFIGURATION STUDY COMPLETE

TIME, 0 MINUTES AND 41 SECONDS
I..END

thesC198

Chromatic and spherical aberration in el



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